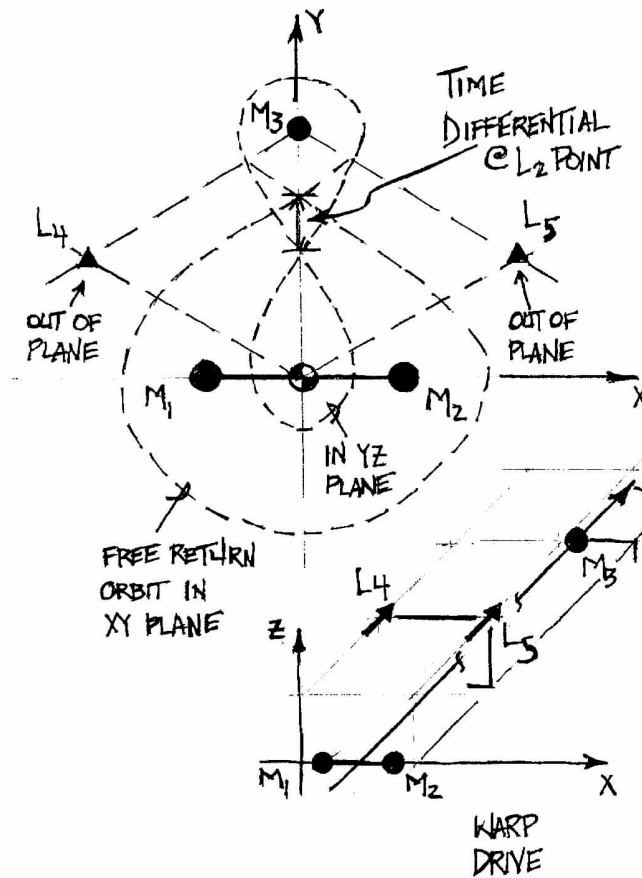


The Conceptual Design Of A Constellation Class Star Ship

WH Clark



A Ph.D. Dissertation

Doctor of Philosophy in Engineering

THE UNIVERSITY OF TEXAS AT AUSTIN

December 15, 2004

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00 Organization

The research is divided into five sections. Each section begins with a major report and is followed by individual papers focusing on specific parts of the major "section" paper. These small reports, or "tabs" are numbered sequentially through the text from 1 to 32. They are intended to present technical journal/conference type papers or posters with solid data and evidence. The major "section" papers are more theoretical.

The numbers in parenthesis throughout the text refer the reader to the detailed mini-papers (and on occasion to the "section" papers, denoted ONE, TWO,...FIVE), as footnotes are typically used.

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0 Welcome

The purpose of this project is to show the existence of two different planes of reference for the solar system, the usual angular momentum plane - plus a new hyperplane about which the motion of all the planets is symmetric - the invariant plane and the symmetric plane, respectively.

There are five sections which have different material supporting this hypothesis:

ONE - the physical evidence

TWO - a computer validation

THREE - real vs. complex planes

FOUR - analogies from other sciences

FIVE - mathematical concepts

All this material implies that the two planes do in fact physically exist, and have forces and an infrastructure associated with them. The planes are therefore the consequences of forces, and so they act as a static or fixed wave front in space. In dynamical theory the intersection of two wave fronts - where the two planes intersect in space - forms a third wave front called a shock wave.

To put this all into context, consider the planes in space with respect to three unique reference axes like a Cartesian Coordinate system. Relativity Theory describes what happens at one point on the intersection of the coordinate axes, and the line that exists as the actual intersection of the planes in 3D space are gravity strings a la "Gravity String Theory." Neither of these bodies of theory can exist without two unique planes in space, is all that is assumed here. Otherwise, you will see that Relativity and Gravity String Theory are both special cases of the classical Problem of Three Bodies.

National Aeronautics and
Space Administration

NASA

Lyndon B. Johnson Space Center
Houston, Texas
77058

MAR 23 1982
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MAR 23 1982

Mr. William H. Clark, II
Route 2, Box 388 D
Leander, TX 78641

Dear Mr. Clark:

We are returning your "Journal of 21st Century Technology" and appreciate your allowing us to review your scientific paper. As you know, the Johnson Space Center's activities are almost entirely devoted to the development and flight of the Orbiter vehicle--a pretty straight forward concept which utilizes existing aircraft and spacecraft technology to build and fly an aerospace vehicle. However, we do some advanced technology development within our Program Development Office, and it was to that organization that I directed your paper.

Your work was reviewed independently by both an engineer and a physicist. Although their backgrounds and scientific/engineering interests are different, both came to the same conclusion. Your concepts about the Unified Field Theory appear to be so profound that you are years ahead of the present scientific thinking in the areas in which you delved.

Given our present financing and direction from Congress, NASA does not currently have the ability to investigate your proposal in greater depth. We do thank you for your interest in manned space flight, and wish you the best in your future endeavors.

Sincerely,



Glen E. Brace, Chief
Personnel Services Branch

Enclosure

00 Project Introduction and Summary

ABSTRACT

The gravity model for the Earth is solved by matrix and statistical methods. Being able to put the system in matrix form means it is a linear system, and the eigenvectors are linear independent and orthogonal - this is the nature of the Fourier Series used to create the model itself - and they are also subject to the superposition of forces. The existence of eigenvectors and eigenvalues implies discrete allowable energy levels. All of this is in the time domain. The fluxions (phasors) developed later show it all in the frequency domain.

Scientists have iterated the equations of motion for the solar system forward in time millions of years and have found the whole system to be completely stable and predictable. Each planet and moon was modeled, in these simulations, as a point mass - e.g. each body acting as though it's entire mass were concentrated at a single point in space.

Contrast this notion to what we know about Earth from satellite geodesy. This science models the gravitational field of the Earth as the infinite sum of a Fourier Series. The matrix of these values has over 100,000 elements and requires the largest, most powerful computers in the world to solve.

It should be disconcerting, how these 100,000 parts of our crude simulation of the Earth's gravitational field - when you put them all together and sum their cumulative action upon other bodies in the solar system, it is as though all the mass were concentrated at a single point.

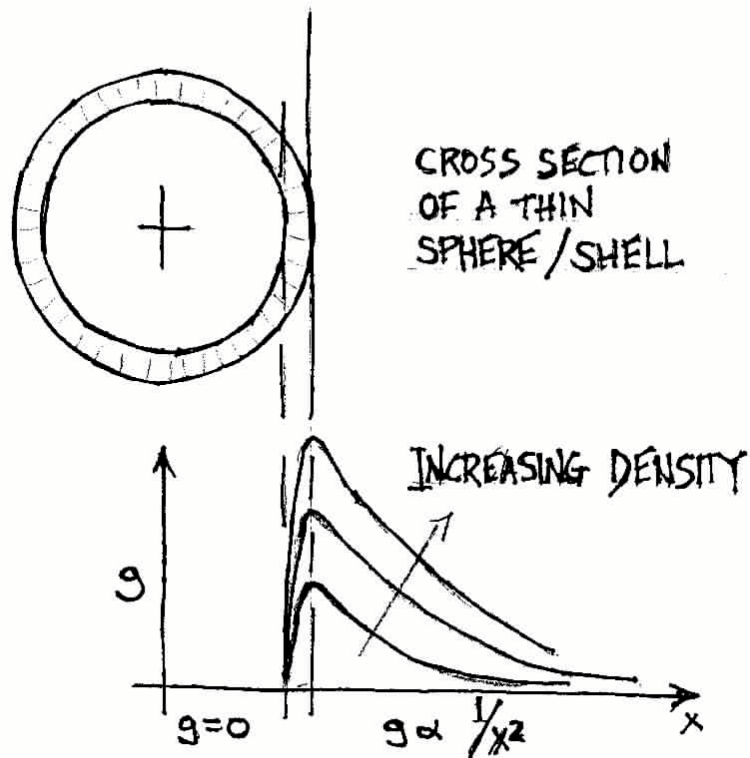
A similar phenomena is exhibited by the equations of motion of N bodies, which reduce down to the simple equations for incompressible flow in fluid dynamics. (11) The question arises, how was it that the solar system evolved from a huge cloud of cosmic dust into a star, nine planets, and many moons - all of which are quite physically complex, but infinitely simple at the same time. Just consider all the different minerals, elements, rocks, and disparate layers of Earth when summed together are so distributed as to have the sum total affect of being perfectly balanced versus a single point the size of the period at the end of this sentence.

Then consider that this same dynamic exists for the sun, the other eight planets, and all their respective moons, rings, and asteroids. More over, the N Body Problem says this also happens on a much larger scale - with vast collections of whole star systems.

The purpose of this investigation is to offer a theory about how this intricate, but completely balanced, organization of matter came to be as it is. This introductory paper is intended to discuss some rudimentary concepts needed to comprehend the research.

Gravity

An interesting aspect of gravity is that the internal behavior of a thin perfectly spherical shell of mass of constant density is the same as though all that mass were concentrated at one point, the center of the sphere. Thus, an infinite number of concentric shells, each of different mass, all assembled in a perfectly spherical planet or moon would behave like a point mass. The same applies for concentric elliptical homeoids, i.e. a thin shell between two similar ellipsoidal surfaces similarly placed.



Gravity of a thin sphere

The crust of the Earth, at least, has no such consistency. Moreover, the Earth is far from spherical, being more pear shaped and with a dramatic equatorial bulge. This makes it even more improbable that the whole planet Earth behaves just like a point mass.

This research will show how this whole intricate organization of different densities of mass in the planets happened, because of the way the original cloud of cosmic dust was organized, then compressed into matter of varying densities. Tesseral harmonics prove this phenomena of concentric spheres zeroing out, as shown later.

The transformation of mass from a swirling eddy of cosmic dust into spinning planets - first gas giants like Jupiter, then eventually solid planets like Earth or Mars - is a dynamic process. Any theory of cosmology that delves into the fine details must be derived from established dynamical theory.

Rigid Body Dynamics

The Three Body Problem (3BP) of Celestial Mechanics has the generality and versatility to encompass this cosmological process. The 3BP is a well known problem of mathematical physics, with many common, easily verifiable applications in solar system astronomy. In fact, the 3BP is so ubiquitous as to be much more than a mathematical curiosity but practically a physical law.

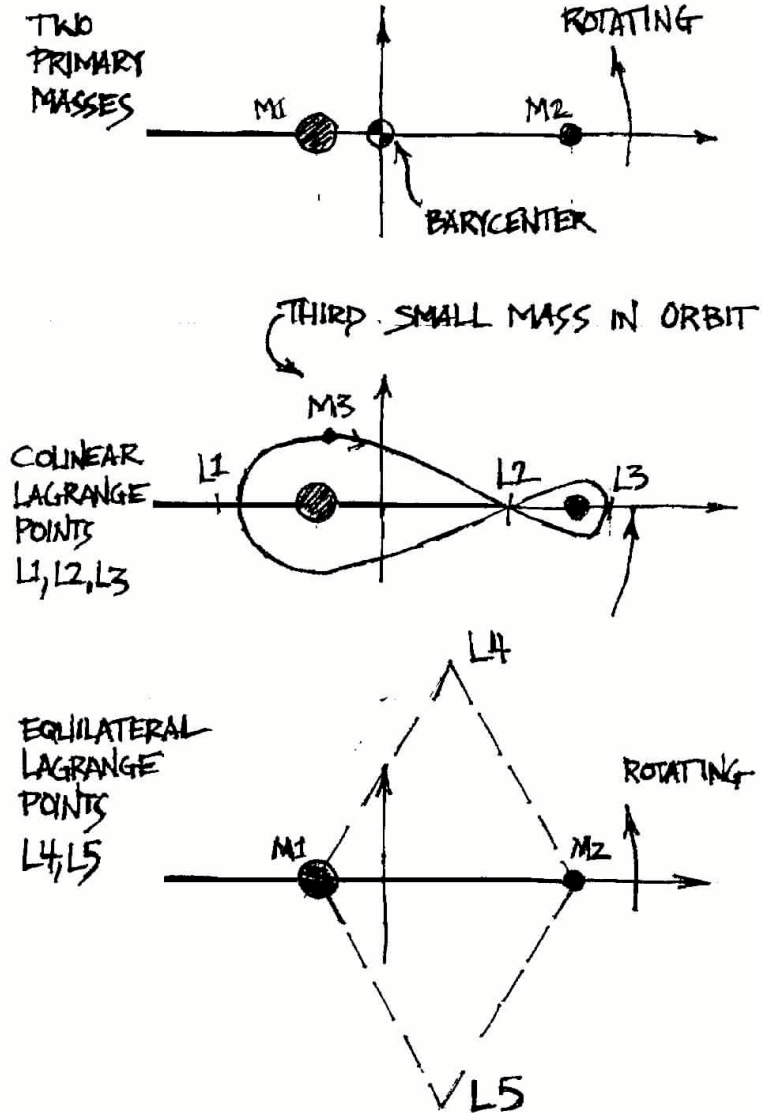
The difficulty in elevating the 3BP to the status of Law is that the general solution is not known in a closed form. The 3BP can be solved in specific situations such as circular coplanar orbits, but just not as the general 3D solution with random motion for all three bodies.

This research adopts the premise that there is in fact no exact mathematical solution to the general 3BP in closed form. However, the laws of nature act in such a way as to drive dynamical systems into known configurations of the 3BP - at which point the constituent bodies reach a dynamic equilibrium, and thus a stable long term periodicity of motion. This is nothing more than a restatement of the well known scientific principle that natural systems seek dynamical equilibrium and stability.

The Three Body Problem

The 3BP of Celestial Mechanics is the study of three bodies in space under the influence of their respective gravitational forces. Typically, two of the bodies are much larger

than the third, and are situated on an axis rotating at a constant angular rate such that the two large bodies remain on this axis.



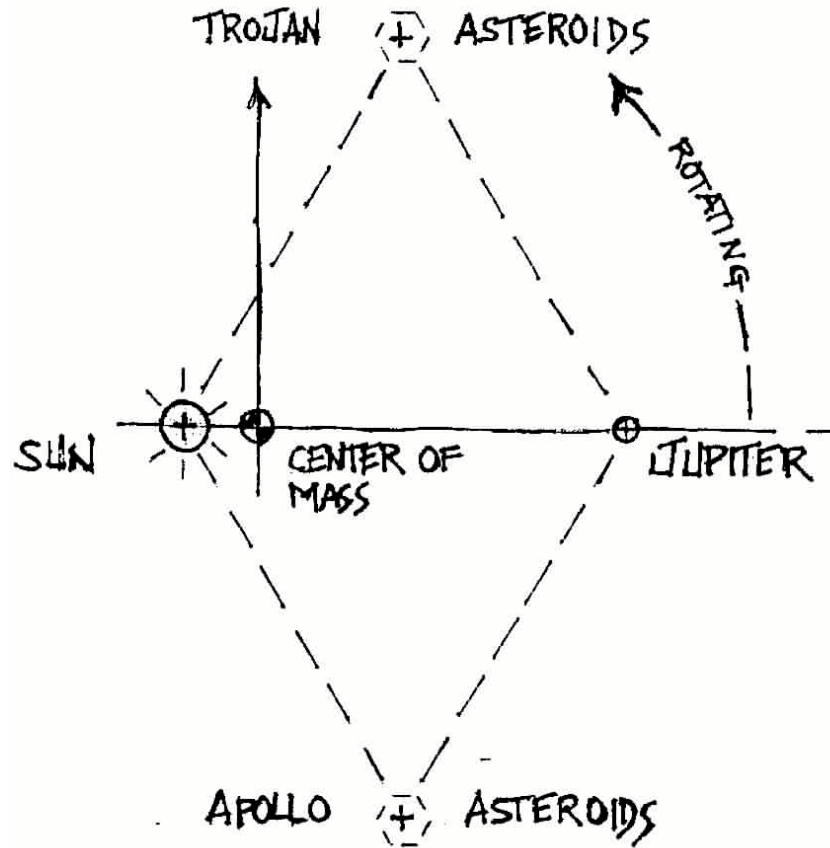
Lagrange Points for the Three Body Problem

The center of mass or barycenter for the two large bodies is at the origin of the coordinate system. A common study of the 3BP is the circular coplanar problem, in which m_2 is in a circular orbit around m_1 (e.g. the sun and Earth), in which case the two bodies in the rotating coordinate system are in circular orbits around barycenter.

The 3BP itself is a study of motion of the third small body. In the simplest case, for the circular coplanar (all motion is in a single plane) 3BP, there are several important features called Lagrange Points, labeled L1 through L5.

The points L1, L2, and L3 are along the m_1 - m_2 axis and they are all unstable equilibrium points. If the third small body is placed at one of these colinear points it remains there, although the slightest perturbation or external force will cause the body to quickly move away, never to return. However, if the small third body is put in a small orbit around one of these points (called a halo orbit) perpendicular to the m_1 - m_2 axis and to the plane of the page, it will remain there indefinitely in a stable orbit.

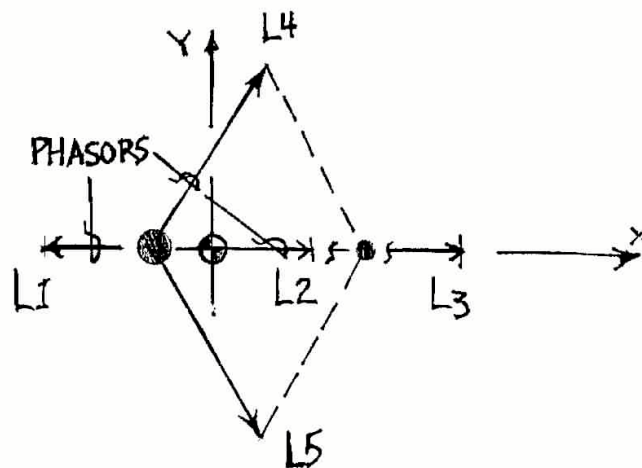
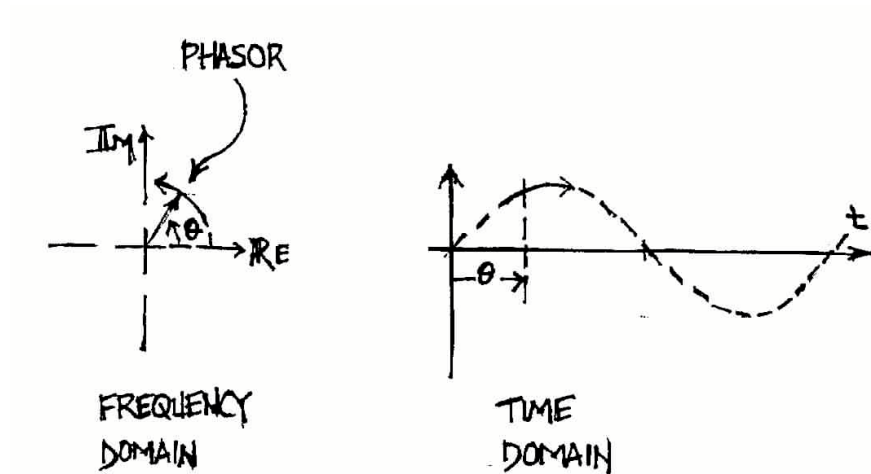
The L2 Lagrange point has another interesting feature. There is a figure 8 shaped orbit with L2 at the cross point between the two bodies. Note that all of these phenomena exist in the rotating coordinate system - called a "free return" orbit. This closed "free return" loop was used by the Apollo moon missions (e.g. an equilibrium solution for the Earth-moon system) so that if anything happened in transit, the spacecraft would always return to Earth. Apollo 13 lost power on the dark side of the moon, and fortunately the space capsule was in such a "free return" orbit that got them back home - or at least close to Earth - without any further thrusts. Notice that this kind of orbit is the only orbit (other than an orbit very far away from the masses) that is inclusive of both large masses, that is stable.



The Sun-Jupiter-Asteroids Three Body System

Two more Lagrange points, L4 and L5, are at the apex of an equilateral triangle with the two large masses as one side. These are stable equilibrium points - i.e. a small body located at the L4 or L5 point will remain there, even in the absence of perturbations. Moreover, small orbits around these two equilibrium points are also stable. In the sun-Jupiter system, the L4 and L5 points are occupied by small clusters of orbiting asteroids, the Trojan and Apollo Asteroid groups.

The mathematics proving these concepts is straightforward. However, the general 3BP itself has no solution. There are solutions to many other restricted cases - e.g. elliptical coplanar motion - but no solution has yet been found for the 3BP with no restrictions on the motion of the three bodies.



Phasors in a Rotating Coordinate System

Dynamically, particles near L4 and L5 are free to move anywhere in the xy plane. Consequently, the forces must be strong indeed for bounded motion at the equilateral points to exist (i.e. gravity is balanced by centrifugal forces near L4 and L5 so that the sum of forces in the rotating system is zero).

The notion of a rotating coordinate system is not the customary way of looking at things. However, nature is in constant motion and actually it is more irrational to assume that natural laws all operate in a fixed, static reference frame. Rotating coordinates are much more natural. That being the case, then centrifugal forces at L4 and L5 are irrelevant and

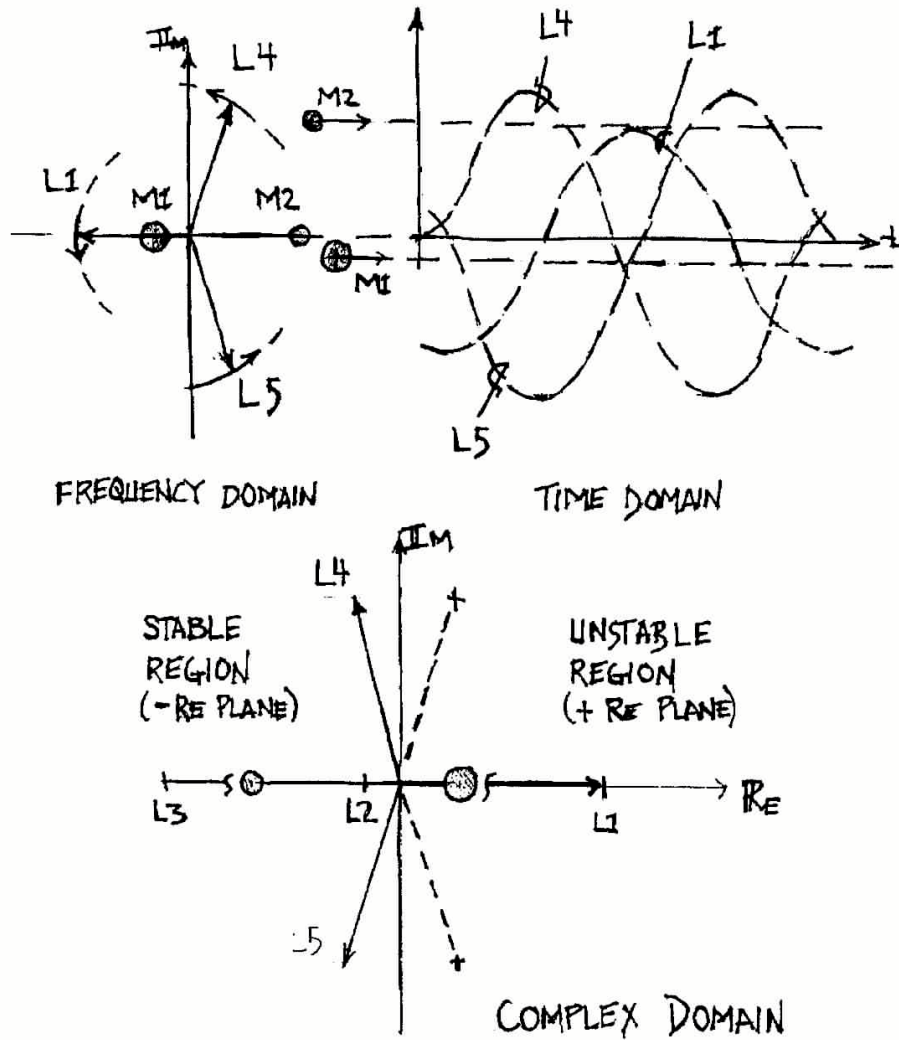
there must be some real forces acting there to keep objects in orbit there such that L4 and L5 are the foci of elliptical halo orbits (actually, not at the foci, but at the center of the ellipse).

Frequency Domain ~ A Rotating Coordinate System

A rotating coordinate system often used in dynamical system analysis is called the frequency domain. In the complex plane, $e^{i\omega}$ is a point on the unit circle rotating at a constant rate ($\omega=1$ for the typical restricted Three Body Problem). The starting angle for motion is called the phase angle (notice the project of the rotating phasor into the frequency domain is a sinusoidal wave). The two equilateral points in the phase domain are represented by phasors. Likewise with the colinear points.

In complex analysis, points to the left of the imaginary axis are stable, those on the axis itself are quasi stable, and those to the right of the imaginary axis are unstable. A more consistent representation is shown below, which uses the reflection principle of the Laplace equation and/or the complex domain. The plot is similar to the three branches or phasors in three phase power systems, with the characteristic out of phase sine waves.

As posed here, the L1, L2, and L3 points would be on the real axis and L2 and L1 would be unstable. The only configuration that satisfies this is the free return orbit that goes through L2 and close to L1 and L3. Notice that a halo orbit at L4 or L5 forms a carrier wave to the respective phasors.



Lagrange Points of the Three Body Problem

I. A General Solution to the N-1 Body Problem

ABSTRACT

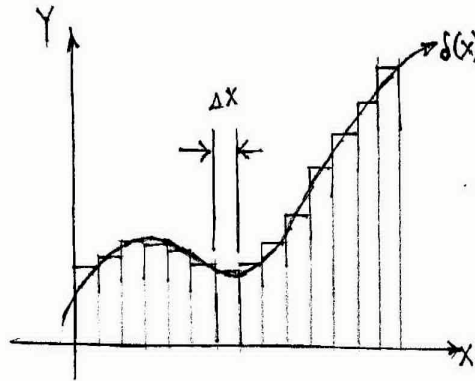
An elegant sweep of solar system cosmology shows how planets were formed from the original cosmic gas cloud. Local eddies of gas coalesced then gradually became solid mass by increments represented by specific Fourier Series expansions. This exact mechanism is silhouetted in how Fourier Series expansions are used in satellite based geodesy to model planet gravity fields, like rings of a tree, and shows how geodetic summations correlate closely to specific aspects of the planet's orbit around the sun. The model posed here, which includes mechanistic explanations for major relativistic phenomena, leads to the dramatic conclusion that Fourier Series are in this instance more than a convenient mathematical tool but a physical representation of the allowable energy states for the major bodies in the solar system, analogous to allowable energy states in the atom. Similar correlations may exist in other Fourier Series models.

The Generalized 3BP

It is widely believed that the Three Body Problem (3BP) of Celestial Mechanics has no general solution. It will therefore come as a surprise to experts in mathematical physics that such a solution is presented here, in heuristic terms and without the use of any analytical tools. The alleged solution is shown to exist in nature, and this solution implies that the 3BP is a stable organization in nature - so much so that natural dynamical systems are driven into the circular coplanar configuration of the 3BP that mathematics has shown to be the most stable solution. In deference to extant analytical theory, the assumption is made herein that nature takes a rather devious approach to solving the generalized 3BP, in such a way that the overall system behaves like the most fundamental restricted 3BP to which a solution is known; in this case, the restricted circular coplanar problem.

This phenomena by which nature solves the 3BP, albeit subtly, takes place in the natural order of the solar system. The general 3BP occurs in a coordinate system local to the planet, whereas the circular coplanar restricted solution - presumably the most stable configuration over the long term - happens with respect to the coordinate system of the entire solar system. The latter inertial reference plane is the hyperplane (2) that is perpendicular to the angular momentum vector of the solar system, which is typically defined in two body

motion as the orbital plane. Simply put, the generalized 3BP is shown to be a regularized solution in this dominant plane of the solar system, or mathematically a hyperplane in which upon which all motion is stable.



- AS ΔX VALUE INCREASES
- ERROR IN AREA INCREASES

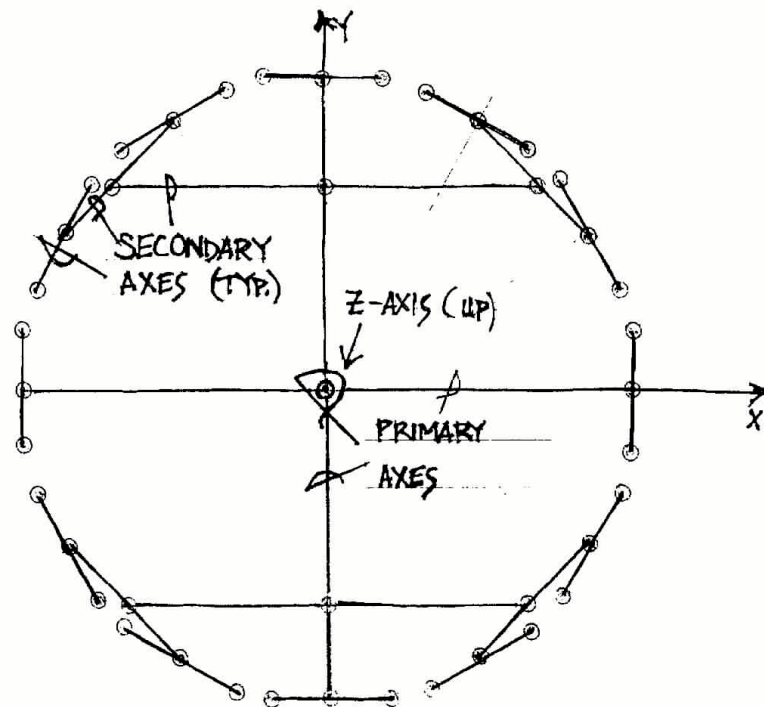
Riemann Summation for Integrals

This transformation happens at discrete intervals or energy levels, going by the Three body Problem in quantum mechanics, where energy can exist only at discrete energy levels. Thus, each stage is translated to a stable hyperplane solution (2); then the next building upon that solution; and so forth, until many separate bodies are involved. (4) This process can theoretically continue indefinitely, until of course N bodies are done, at which time the whole configuration reduces down to the equations of motion for an incompressible fluid. This report shows how the summation happens for increments of inclination in the orbit, simultaneously as a similar process happens for increments of eccentricity of the orbit. The process is a little different in either case, but the result is always a circular coplanar restricted 3BP which has a known mathematical solution. (Note in the above figure, the limit is the Delta Function such that the larger $\delta(x)$ the less accurate the integral.)

It will come as a surprise to experts in Celestial Mechanics, just how easily nature solves the generalized 3BP, and how elegantly the solution lends itself to the mathematics. This notion will immediately rankle skeptics, and it is in deference to their doubts that the solution will here be posed in a cosmological context. The explanation begins when the solar system was nothing more than a cloud of gas, and shows how planets were formed and how they came to be as they are today (1) - with particular reference to their orbital elements

versus their gravitational properties; e.g. the distribution of mass, and its density, within individual planets.

It will be shown that the 3BP played an important role from the very beginning. The 3BP is more than an interesting mathematical problem, but the embodiment of a physical law of nature. The 3BP represents - at least in its stable and known solutions - a fundamental level of dynamical stability in nature, to the extent that natural motion is driven to be organized in such a scheme. The unsaid assumption in this hypothesis is that, given there are many versions of the 3PB with known solutions, each one of these represents a step in the evolution of a chaotic mass of random particles of matter into a consistent planet in a stable orbit around a central body. These problems, sequentially, form together an envelope curve (27) that is itself another solution to the 3BP.



Walker's N Body Problem

This report makes the leap from chaotic gas cloud directly to the most stable 3PB of them all, the circular restricted coplanar 3BP, accepting the fact that there are many additional steps in between via quasi stable orbits, but here for purposes of brevity taking the overall perspective of the process. Experts in mathematical physics have already derived all the intermediate steps from one stable circular state to the next. Thus, far from posing a

solution to the generalized 3BP on independent merits alone, this report shows that the general solution in fact involves the entire body of theory on the 3BP, with transitions between each quasi stable orbit happening perhaps by virtue of the various non-standard orbital elements. These details are beyond the scope of this report and, in any event, are moot if the basic process itself is not first established.

In the Beginning

For some reason each nascent planet was forced slightly out of the orbital plane of the sun-Jupiter system, and the planets went from gas to mass in order to maintain a dynamic equilibrium with the whole. These changes not only transformed gas to solid matter, but organized the matter by density in such a way that the gravitational field can be modeled by Fourier Series - as done in the science of satellite based geodesy.

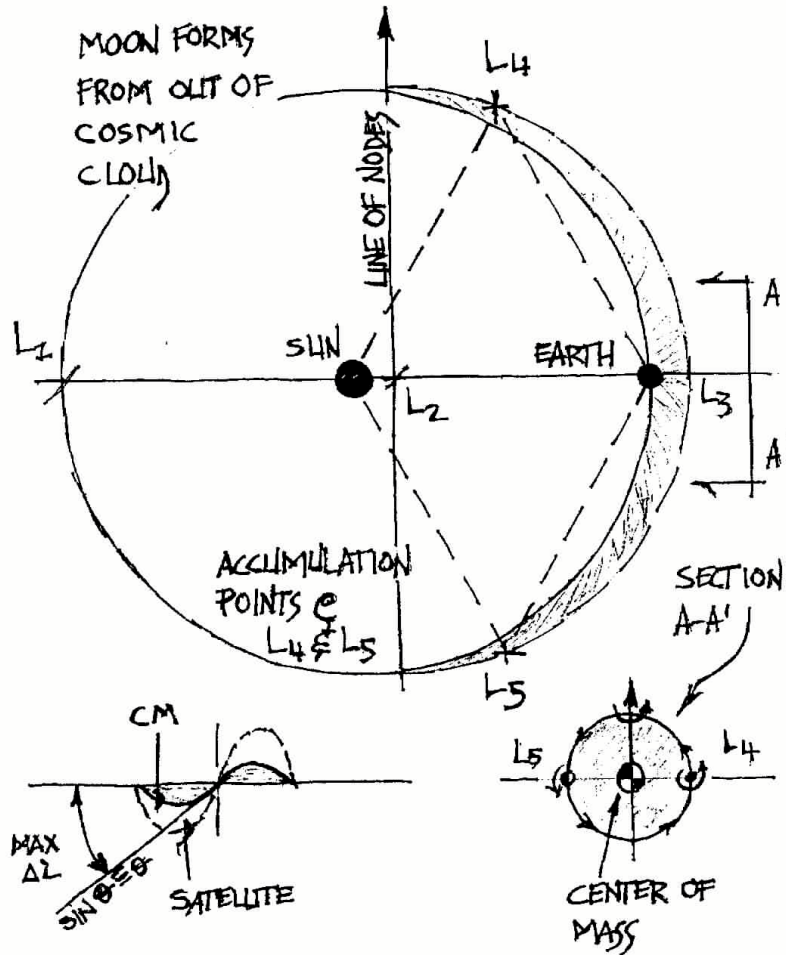
In the next few pages you will see how the divergence of a planet's mass from the invariant plane transpired in discrete steps, each one a single term in a Fourier Series. The Fourier Series we now use to model a planet's gravitational field is a permanent record of these changes, like rings in a tree, solidified in the rock of the planet itself.

The important question now is, why did the small eddies of gas that were to become planets diverge from the invariant (i.e. angular momentum) plane at all? Relativity specifies a slight warping of space in the vicinity of a high gravitational field, such as the sun. In this case, however, the plane about which the mass was distributed could have had local discontinuities or bumps, projecting above or below the orbital plane formed by the summation of the whole solar system's cloud of gas, into a single common angular momentum vector. A facsimile of these local variations will be derived shortly.

The invariant plane as we now know it is dominated by Jupiter, which is by far the most massive planet. Before the formation of Jupiter from out of the gas cloud, all the mass in the solar system would have been roughly symmetrical to the invariant plane. Indeed, all mass in the solar system still moves perfectly symmetrical to such a "symmetric hyperplane," with the sun at (near) the barycenter. (2)

Presumably this symmetric hyperplane is the same as the one that existed aeons ago when the solar system was all just gas. As for the force or phenomena which caused individual eddies of gas (future planets) to diverge from this symmetric plane, perhaps a pattern in the distribution of the planets in a "system wave" (or "system surface" as the case may be) in the current epoch offers an explanation. (3)

The actual process of moon formation is kind of an hysteresis phenomena in the rotating coordinate system. Refer to (14) for they dynamics of how debris are collected in the rings of Saturn - a precursor condition to moon formation.



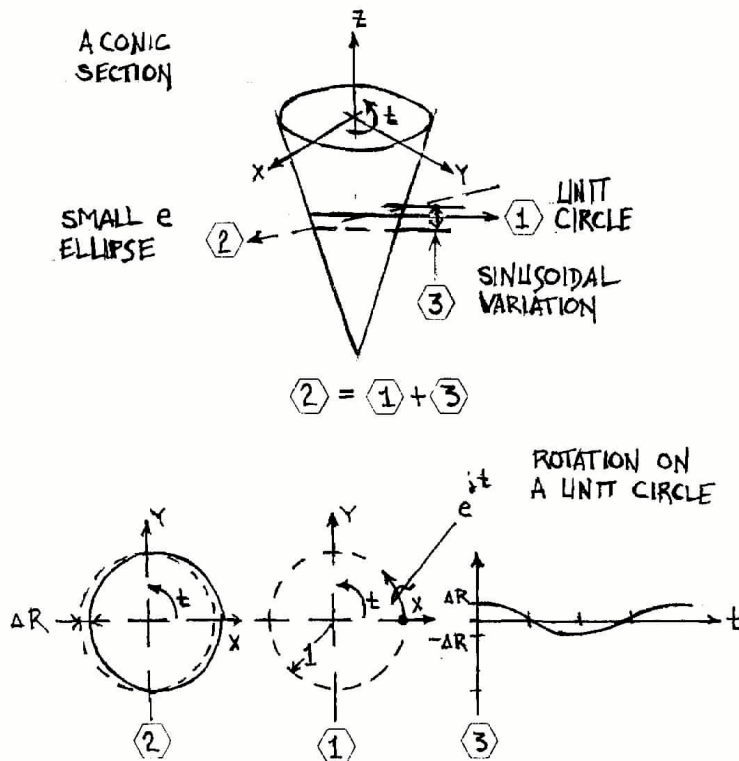
"A Time Before the Moon Appeared" ~ a Myth or Reality?

The "system wave" is a study of the motion of the planets with respect to the ecliptic plane. The constraints for finding this helical 3D wave (longitudinal wave in 2D) are (i) a planet moves such that it is attached to the wave so that it's motion up/down in the course of a revolution is due to the inclination of its orbit; near/far on the wave is due to the eccentricity of its orbit; (ii) the planet's axis of rotation is perpendicular to the wave such that (iii) during a revolution this axis points always to the same place on the celestial sphere, like Earth and the North Pole. A helical wave was found that satisfies these criteria for all the planets such that

each planet occupies a specific segment of the wave - a stable equilibrium position for the Ten Body Problem (10BP) - the point being that this wave pattern could have caused the small eddies of the primordial gas cloud to be pushed above/below the original plane of symmetry, e.g. the common orbital plane for the solar system gas cloud as a whole. Presumably this "system wave" is caused by galactic forces external to the solar system, as some aspect of the gravitational phenomena holding the entire galaxy in its specific configuration. This notion is discussed at length in other sections.

Mass from Gas

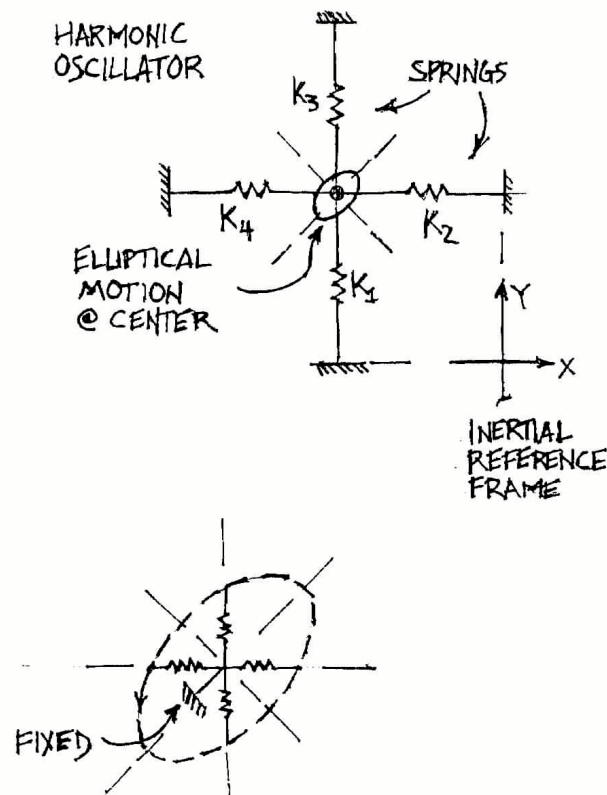
Now to consider the dynamic mechanism by which the planets became solid matter from out of a cloud of gas. Please note that this is not a trivial exercise in logic because, as you will see later, it silhouettes the opposite process - i.e. the conversion of solid matter to gas or plasma; a.k.a. atomic fission, a promising power source.



Conic Sections vs. the Small Eccentricity Ellipse

The following analysis is predicated on the assumption that the solar system, from beginning to end, is a single dynamic system and that each constituent part of the solar system is compelled to - within its physical limitations (which are nominal for a gas giant, but much more constrained for a solid planet) - seek a stable configuration. The strongest common force for the gas cloud solar system was that it was rotating in a single plane. Thus the organizing force or criteria was angular momentum alone and the constant direction (defining the plane and by this analysis the orbital inclination) and magnitude (defining the "strength" of the plane and by analogy the orbital eccentricity) of the angular momentum vector.

A simple study in geometry shows that an elliptical orbit in one plane can be modeled by a circular orbit in an adjacent plane at a small angle to the first one, by a Fourier Series type approximation. (4) This is easy to show in rotating coordinates (above). Observe that a sine wave is the projection of a point fixed on a rotating circle.

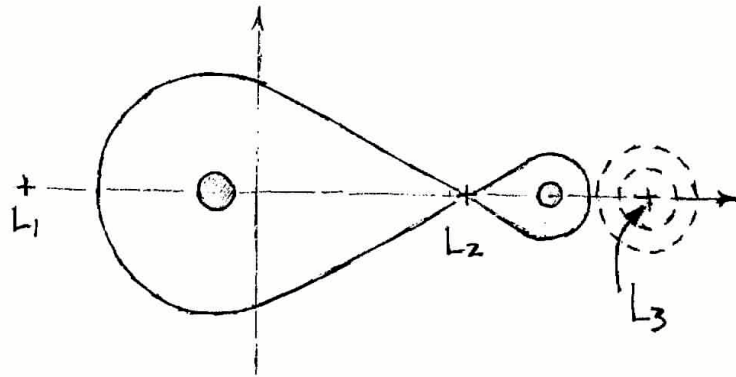


The Two Dimensional Harmonic Oscillator

You can get an exact fit to the (ellipse-circle) by:

- (1) Varying the rate of rotation of the circle (i.e. time) = **relativity**
- (2) Assume there are more terms to the Fourier Series approximation = **celestial mechanics**

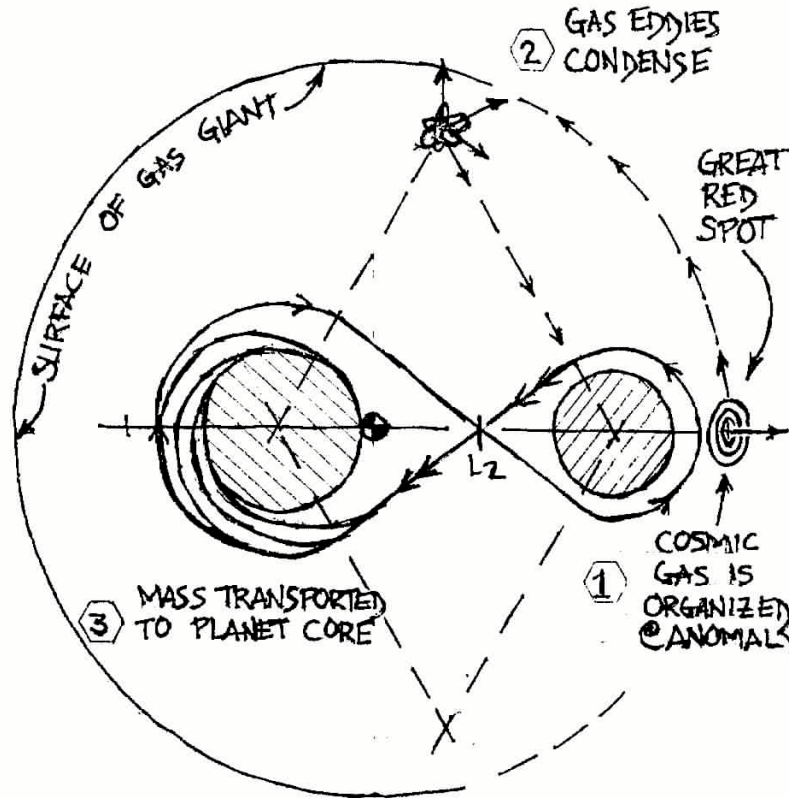
Notice in the illustration that the displacement of springs is the same in the bottom case as in the "direct" case at the top of the illustration. The harmonic oscillator in two dimension produces motion in the shape of an ellipse (also in the 3D case) where the forces are $1/r$ forces (because the center of the ellipse is the focus); e.g. the same as with springs, with the angular frequencies in the x- and y-directions being equal. Incidentally, this is just how a "halo orbit" works - small ellipses centered at the Lagrange Point, in an orbital plane inclined to the coordinate axes of the rotating system.



Jupiter's "Great Red Spot" as a Halo Orbit

Posing this as a 3BP, with the large second primary a gas giant of constant density, then these results are as stated and the analysis is valid in the present situation without loss of generality. The conclusion is that the whole body of the planetary gas cloud moves (or is compelled to move, if you accept the "system wave" hypothesis) away from the symmetric plane of the solar system. Once all the matter in the cloud is "solidified" the process stops, leaving the planet in just as perfect a dynamic equilibrium with the solar system as a whole, as it was in the beginning.

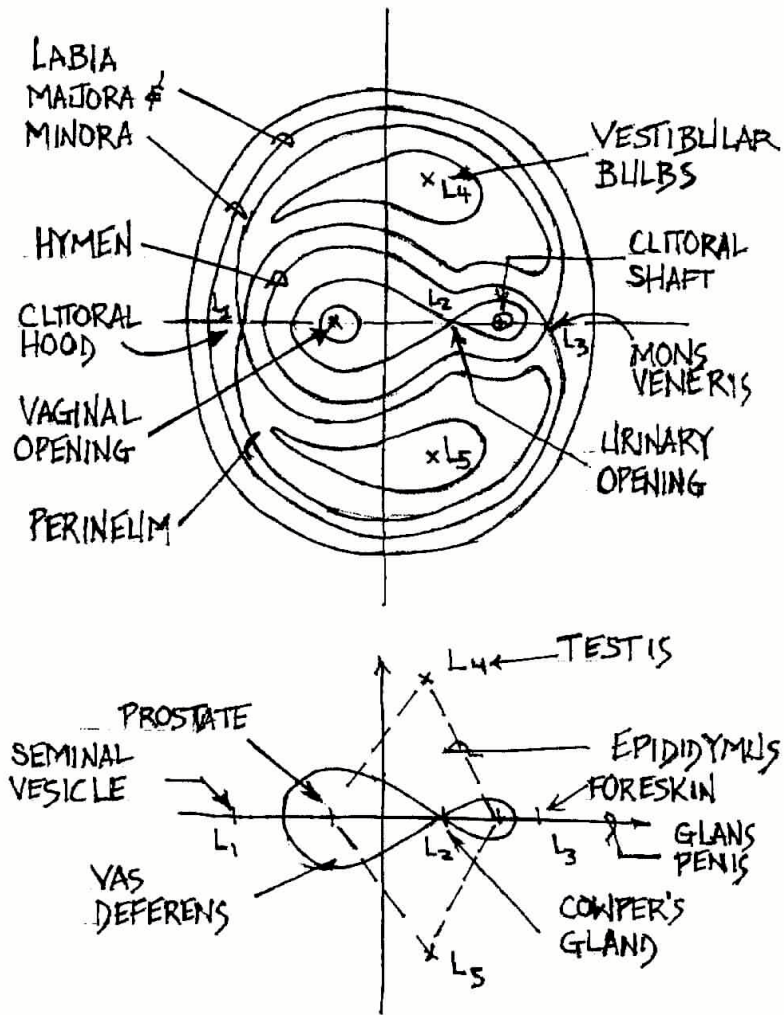
It is worth mentioning that the "Red Spot" on Jupiter may represent this ongoing process, and also the precession of Mercury's perihelion - Mercury being such a small planet, and unable to solidify since it is so close to the sun, the balancing mass has to be within the sun itself. (5) Later the analysis of tesseral harmonics correlates with Jupiter's bands of moving gas.



Jupiter's Great Spot and the Process of Planet Formation

Most scientists believe the precession of Mercury's perihelion is "caused" by Relativity. However, there are still many who adhere to the equally plausible explanation of the precession being caused - in a process akin to that caused by Earth's equatorial bulge - by a specific density anomaly within the sun. The problem with the latter explanation is that it offers no explanation for the bending of light by the sun's intense gravitational field, but (5) may rectify this issue.

This raises the question, could gravity within a star or gas giant planet be proportional to distance or $1/r$ and not $1/r^2$? In such a force field, elliptical orbits are centered not at a focus but at the center of the ellipse, which would be a more logical place for it to be in some circumstance; e.g. motion of an anomaly within the interior of a star. (Moulton)



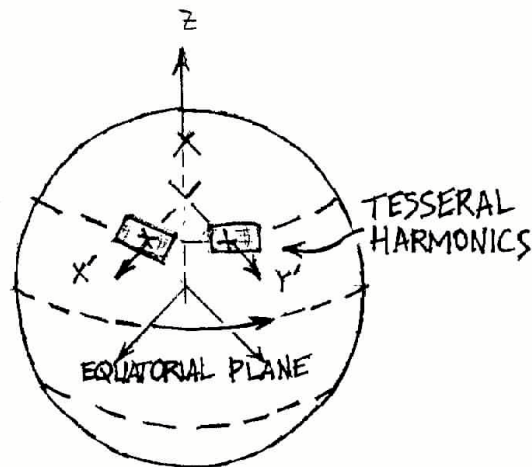
The Three Body Problem of Human Physiology

This would imply a specific transition between the original $1/r$ universe and the current $1/r^2$ universe - and, given that Jupiter and perhaps the sun are remnants of the original $1/r$ universe - there must remain some boundary region between the two. This would fit the notion of weak and strong gravitational forces. This suggests that "black holes" might not be the drastic discontinuities specified by Relativity Theory, but simply transitions to the next level of reality, e.g. different powers of r than $1/r^2$.

The geometry of the $1/r$ and $1/r^2$ systems is that they are equivalent for circles. Ellipses are a combination of sinusoidal waves, circles being ellipses of $e=0$; and so waves

emanating from the separate systems can coexist as in a 3D EM wave. That is, elliptical orbits exist in a $1/r$ system, only the central body is at the center of the ellipse and not at a focus, as in the $1/r^2$ system. Likewise, the intersection between two such systems would be a helix, the shape derived for the "system wave." Thus the body of theory as referenced in the footnotes () is geometrically consistent.

The one thing missing from this model of Jupiter's inner workings is the motion of mass within the given parameters. The next illustration is a fractal level of the Zero Velocity Curves to human physiology, and the mode of operation of these organs follows the processes described here in planet formation.



Anomalies in the Earth's Gravity Spheroid

Review

The analysis shows that the Fourier Series used to represent a system are more than a convenient mathematical tool of modeling, but a representation of actual physical processes - and perhaps even delineating the allowable discrete energy states for a system so represented. In the case of orbital perturbations about a unit circle, the Fourier Series are even more compelling as an analytical tool, being the equivalent of a Taylor Series expansion for linear systems, with the added bonus of representing actual dynamical processes integral to the solar system so represented. Given that neighboring optimal paths are simply trajectories differing by a Taylor Series from an existing local optimal path, the calculus of

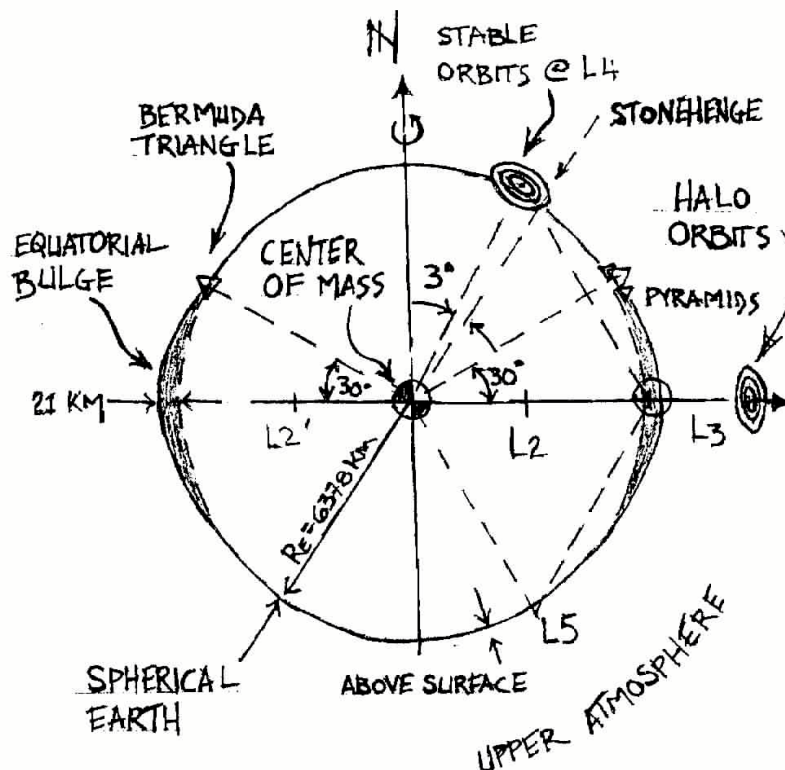
variations shows that the elliptical perturbation from a unit circle orbit is simply the local minimum energy trajectory for the system of particles in question.

The idea of an N-1 (N minus 1) Body Problem is simply to say that if the N Body Problem is nothing more than the equations of motion for particles in incompressible fluid flow (11), then a body such as Mars could be transformed to an N Body Problem by simply adjusting the local distribution of mass - and viola, Mars is awash with oceans of water.

Geodesy

The existence of Tesseral harmonic mass distribution on Earth shows the breakdown in the concentric shells of Earth's uniform gravity spheres. (refer to the analysis of $1/r$ zero velocity curves on the following pages).

Now put all these ideas together, and consider the Earth's equatorial bulge as an anomaly.

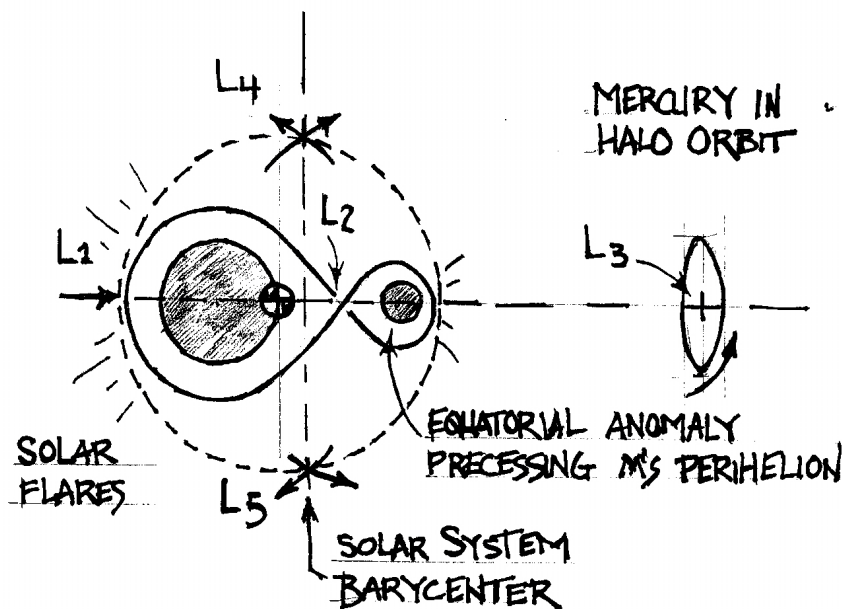


Paranormal Phenomena and the Three Body Problem

In this scenario, halo orbits can exist at L4 and L5 and these are just above the surface of the Earth, L3 is somewhere in the upper atmosphere, and L1 is deep in the ocean. The latter can contribute to long term ocean currents, whereas the others may affect global weather. (It is likely that global warming will eliminate the crop circle phenomena by melting the ice caps and in the resulting distribution of mass from the poles, alter the dynamics of this model.)

The Tesseral harmonic anomalies shown earlier on Earth exist at specific latitudes like Jupiter's "Great Red Spot" and exist as "halo orbits" in the rotating coordinate system, that become manifest as el nino and la nina as long term high pressure regions in the upper atmosphere.

Now consider an anomaly within the sun that has Mercury in a halo orbit around the L3 Lagrange point.



Precession of Mercury's Perihelion

This is possible only in a $1/r$ gravity system, in which the distances between Lagrange Points are much extended. Could a $1/r$ system also exist for gas giants? is discussed later.

It is now possible to derive the Lagrange Points in a rotating 1/r circular coplanar system:

$$F = m\ddot{r} \neq \frac{m_1 m_2}{r^2} \text{ but } = \frac{m_1 m_2}{r} \hat{r} \text{ where } r = \xi + \eta + \zeta$$

Thus,

$$\ddot{\xi} = (1-\mu) \frac{\xi_1 - \xi}{r_1^2} + \mu \frac{\xi_2 - \xi}{r_2^2}$$

$$\ddot{\eta} = (1-\mu) \frac{\eta_1 - \eta}{r_1^2} + \mu \frac{\eta_2 - \eta}{r_2^2}$$

$$\ddot{\zeta} = 0$$

And

$$\xi = x \cos t - y \sin t$$

$$\eta = x \sin t + y \cos t$$

$$\zeta = z$$

Taking the first and second derivatives,

$$\dot{\xi} = \dot{x} \cos t - \dot{y} \sin t - x \sin t - y \cos t$$

$$\ddot{\xi} = \ddot{x} \cos t - \dot{x} \sin t - \ddot{y} \sin t - \dot{y} \cos t - \dot{x} \sin t - x \cos t + y \sin t$$

Rearranging terms,

$$(\ddot{x} - \dot{y} - x) \cos t - (\ddot{y} + 2\dot{x} - y) \sin t \tag{A}$$

$$= \left[(1-\mu) \frac{x_1 - x}{r_1^2} + \mu \frac{x_2 - x}{r_2^2} \right] \cos t + \left[\frac{1-\mu}{r_1^2} + \frac{\mu}{r_2^2} \right] y \sin t$$

$$(\ddot{x} - 2\dot{x} - x) \sin t + (\ddot{y} + 2\dot{x} - y) \cos t \tag{B}$$

$$= \left[(1-\mu) \frac{x_1 - x}{r_1^2} + \mu \frac{x_2 - x}{r_1^2} \right] \sin t - \left[\frac{1-\mu}{r_1^2} + \frac{\mu}{r_2^2} \right] y \cos t$$

Now take,

(A)cost + (B)sint and (A)-sint +(B)cost to get:

$$\ddot{x} - 2\dot{y} - x = -(1-\mu) \frac{x - x_1}{r_1^2} - \mu \frac{x - x_2}{r_2^2} = U_x$$

$$\ddot{y} + 2\dot{x} - y = -\left(\frac{1-\mu}{r_1^2} + \frac{\mu}{r_2^2}\right)y = U_y$$

want something like $U = \frac{1}{2}(x^2 + y^2)$

Now to solve $\int \frac{(x-x_1)}{(x_1-x)^2 + y^2} dx$ Let $u = x^2$ so that $du = 2x dx$ and solve $\int \frac{x}{x^2 + y^2} dx$

$$\Rightarrow 2 \int \frac{du}{u + y^2}; \text{ let } q = u + y^2; dq = du$$

$$\Rightarrow 2 \int \frac{dq}{q}; \text{ thus}$$

$$\begin{aligned} \Rightarrow U &= 2 \ln q + c_1 \\ &= 2 \ln(u + y^2) + c_1 \\ &= 2 \ln(x^2 + y^2) + c_1 + c_2 \\ &= 2 \ln[(x-x_1)^2 + y^2] + c_1 + c_2 + c_3 \end{aligned}$$

where two of the constants are for the offset coordinate system. Substituting,

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1}{2}(1-\mu)r_1^2 + \frac{1}{2}\mu\ln(r_2^2)$$

where the 2nd and 3rd terms for the usual equation are $\frac{1-\mu}{r_1}$ and $\frac{\mu}{r_2}$

So you still have, $V^2 = 2U - C$ and the usual zero velocity curve mapping. Consequently, instead of re doing the whole set of zero velocity curves, it is possible to just do a conformal

mapping of the usual zero velocity curves per $\frac{1}{r} \Rightarrow r \Rightarrow \ln r \Rightarrow \ln r^2$

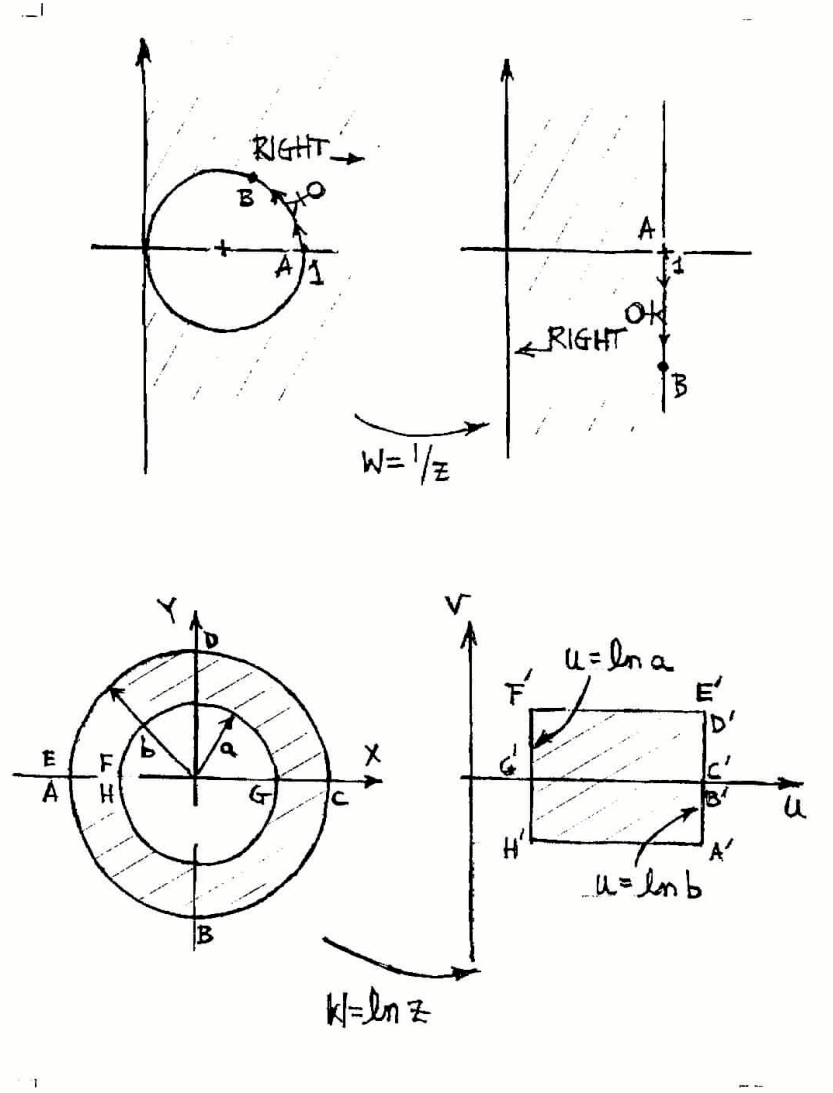
Or,

$$U = \frac{1}{2}(x^2 + y^2) + \Phi$$

and

$$V^2 = x^2 + y^2 + 2\Phi + C$$

The geometry of the mappings is shown below.



Mapping to $1/r$

1 Small Eccentricity Ellipse

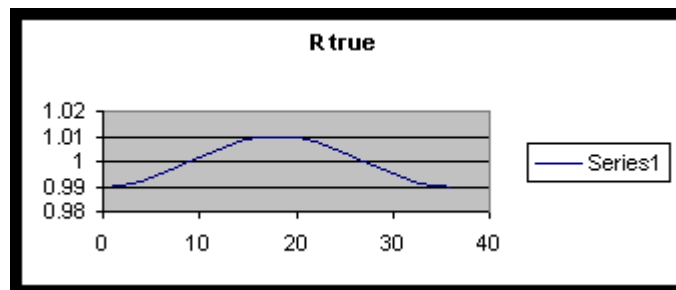
ABSTRACT

It is desired to study of the Three Body Problem (3BP), in an effort to achieve a general solution in closed form. Clearly this is not possible in an unrestricted sense, so an attempt is made to form a general solution to a slight variation of the restricted circular 3BP. This is accomplished by assuming the circular orbit is a unit circle, as is customary in 3BP analyses, and that the orbit is an ellipse of small eccentricity. A small eccentricity ellipse is closely approximated by an off center circle. The purpose of this small paper is to derive an analytical solution for this type of ellipse. The solution is a Fourier Series in cosines that has a rapid convergence. Solutions of this form, in sines and cosines, are especially amenable to 3BP studies and this is shown in the general unrestricted 3BP.

The Unit Ellipse

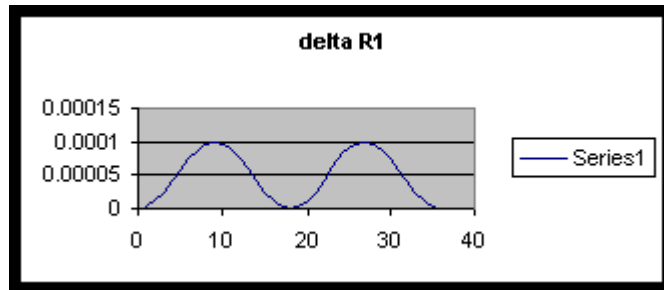
In the late 1800's it was popular to use Fourier Series to represent the orbits of the planets in the Solar System. The technique was suspended by Poincare et al because they believed Fourier Series solutions were unstable, and represented an unstable system. We know much more about the solar system now, enough to have confidence that a seemingly unstable system is likely due to aspects not explained by current theory.

Consider a unit circle inscribed within an ellipse of small eccentricity, with the center of the circle coinciding with a focus of the ellipse. It is easy to see that the variation between the radii varies systematically, if not exactly sinusoidally. In practice, an ellipse of eccentricity .01 varies from the unit circle by a cosine wave of magnitude .01



An ellipse of eccentricity .01 versus the unit circle

The maximum error can be modeled accurately by another cosine curve of half the wavelength, and a magnitude of .00005 And so forth.



An ellipse versus the first term approximation

This is the basis for Fourier Series: any curve can be reproduced exactly by the sum of sine and cosine waves. This is shown above for the case of a small eccentricity ellipse. A similar result can be achieved for a circle in an orbital plane at a small inclination to the original orbit. These two functions are orthogonal and, in fact, could be considered a 3D wave with sinusoidal variations in perpendicular planes similar to the behavior of electromagnetic waves. (29) Interference with another such wave can be made to vary the orientation of the ellipse and the direction of inclination of the orbital plane. All of these properties, which together comprise the orbital elements, arise from Fourier Series variations of a simple unit circle.

The formal equation is

$$r = 1 - \frac{e^2}{1!} \cos\left(f + \frac{\pi}{2}\right) - \frac{e^3}{2!} \cos\left(2f + \frac{\pi}{2}\right) + O(e^4)$$

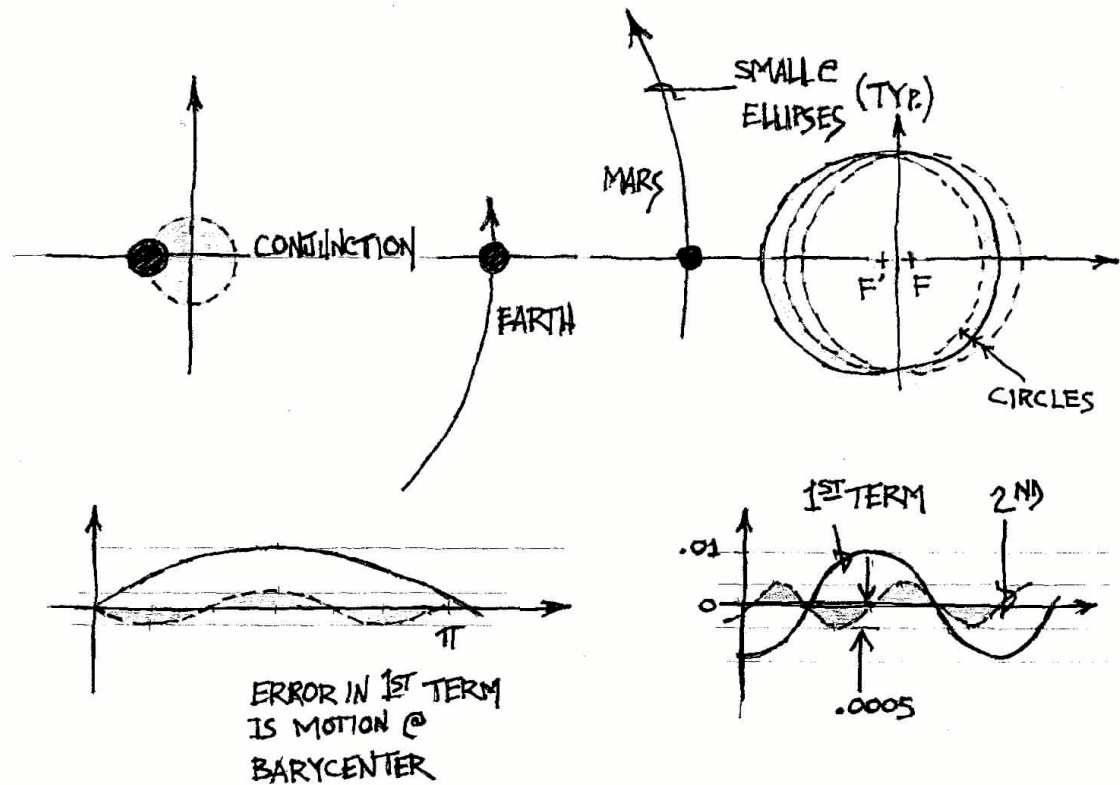
Where the radius from the focus of the central body is accurate to eight significant digits with just two terms, for an ellipse of eccentricity 0.01 This size ellipse is assumed to be the highest eccentric ellipse for which this series representation is viable.

Mean Motion

There are two ways to consider the ellipse as the sinusoidal perturbation from the unit circle. Both schemes account for the orbital motion difference on an elliptical path versus the unit circle (e.g. the 3BP is in rotating coordinates).

In a system of nested increments of eccentricity, the radius vector is originated at the previous level; at the "empty focus" of the current level, so that the angle it sweeps out is the mean anomaly (accurate for small values of e). The overall path, like an envelope curve (27) is the sum of all the incremental e -steps.

In the mathematical treatment of an individual increment, the time differential for travel on an ellipse can be corrected by inclination of the orbit (adding sinusoidal wave(s) perpendicular to the orbital plane). Orbital inclination is irrelevant in the circular restricted coplanar 3BP, thus increments of i and e are added simultaneously, to keep the overall system in dynamic equilibrium. This notion is supported by a study of the motion of the planets in our solar system. (3) Both of these ideas will be developed further in subsequent reports.



Small Eccentricity Ellipse versus Motion of Barycenter

2 The Symmetric Hyperplane

ABSTRACT

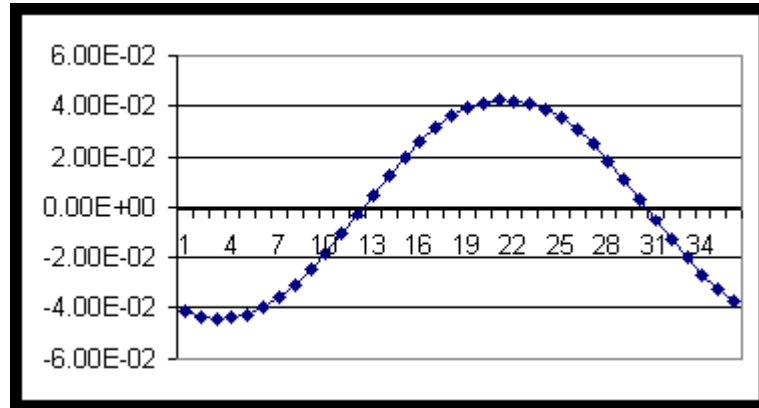
The purpose of this study is to find a plane about which the motion of all the planets is symmetrical. The method used to reach this result takes the form of an inverse problem; e.g. the study of a body of seemingly random data, trying to create a theory to explain it. This is basically what Kepler did; but now a different, perhaps more elaborate, solution is sought. A compelling geometric basis of this symmetric plane is to see that it is formed by a straight line fixed at either end to a point on two rotating circles, one centered at the Sun and the other centered at say Jupiter. This line, while making a complete revolution around the Sun (each circle, meanwhile, making one rotation), traces out the symmetric plane. The resulting shape has the Relativistic space-time distortion at the Sun; and a kind of vortex at the outer pivot point. This paper makes no pretense that the symmetric hyperplane derived has some theoretical basis, other than to provide sufficient facts to justify not disregarding it all together - holding out hope that the elegance of the fit might prove fruitful in using this as a coordinate transformation scheme.

A Geometric Study

A powerful technique, frequently applied in Celestial Mechanics to gain insights into an otherwise intractable problem, is to transfer coordinates to another frame of reference. What is sought here is to find a three dimensional plane in space about which the motion of all the planets in our Solar System is perfectly symmetric. Consider the orbital positions of the planets, reduced to a manageable set of data. The method in concept is as follows:

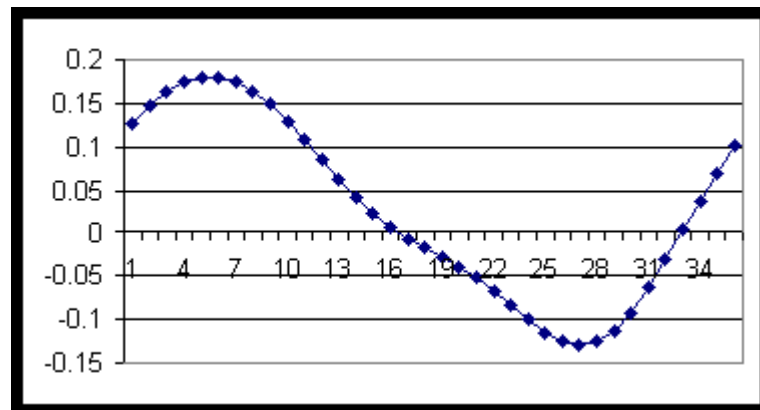
- (1) The position of each planet at 0° heliocentric longitude is calculated from the orbital elements
- (2) A linear least squares fit is made of these nine data points, giving a slope and intercept
- (3) This is repeated at 10° increments through 360° of longitude

The slope of the 36 lines thus created varies sinusoidally, which thus defines a flat XY plane in 3D space.



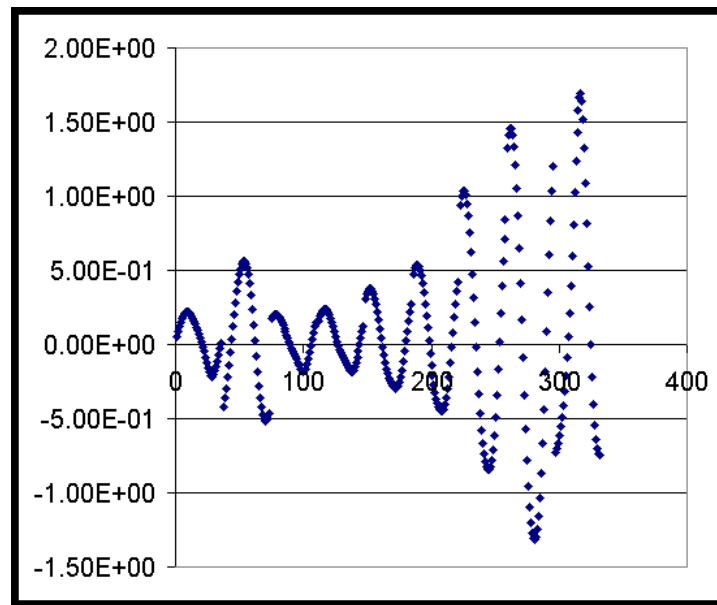
Slope vs. Heliocentric Longitude

The intercept of the 36 lines varies sinusoidally. This creates a distortion at the origin; i.e. in the vicinity of the Sun.



Intercept vs. Heliocentric Longitude

The true measure of this new plane in space is how symmetric the motion of the planets is with respect to it. This motion is shown in the topmost graph to be uniformly sinusoidal in the z-direction.



Position Perpendicular to the Symmetric Hyperplane

Observe that all planets (excepting Venus, and Pluto) are sinusoids exactly in phase, with the same leading phase angle. Venus and Pluto are exactly pi radians out of phase with the other planets; they also exhibit retrograde rotation, so being out of phase with the other planets in prograde rotation is a logical consequence. (The graphs of the planets are arranged in their actual sequence but adjacent to each other, disregarding the large gaps between each orbit.) The curve for each planet is not a closed curve because of the action at the origin of this new coordinate plane, which is not fixed at barycenter, which results in one cycle of a sinusoid, whereas a coordinate system fixed at barycenter would generate an elliptical shaped pattern versus the symmetric plane.

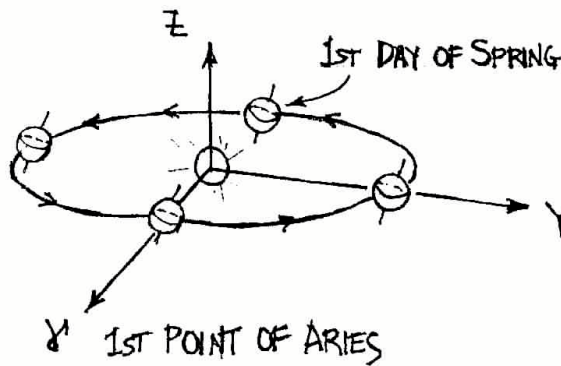
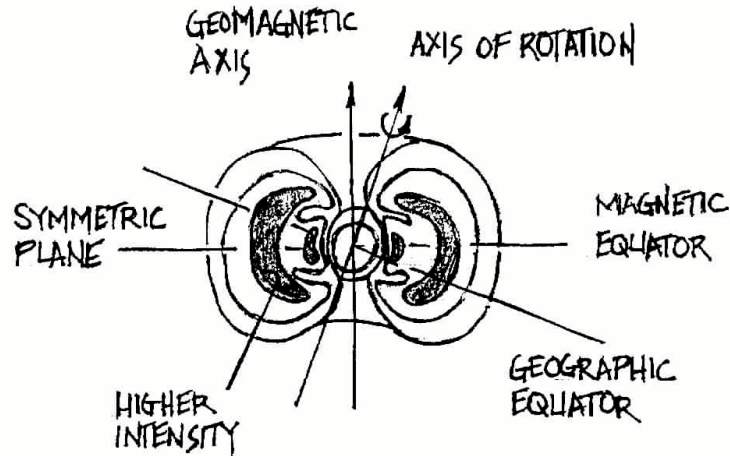
The important thing is that the motion of each planet is symmetric with respect to this plane. In the course of a revolution around the Sun, each planet actually traces out an ellipse in an inertial coordinate system. The geometry of the ellipse determines the inclination and eccentricity of the orbit, and the orientation of the ellipse determines the longitude of the ascending node. In a manner of speaking, there is two body motion in a local coordinate system rotating with each individual planet.

Upon closer inspection, the symmetric plane derived here is not the sort of static plane that is usually associated with the word – that is, it is not completely flat in 3D space. There is a relativistic distortion of space at the origin.

The symmetric plane is not so easy to conceptualize, but consider a straight line affixed in sliding contacts to two circles: a small one with center at the Sun and a larger one

with center in the vicinity of Jupiter's orbit, near to the transition between the small inner planets and the much, much larger outer planets. This line, rotating through 360° in 3D space forms the symmetric plane; both circles also rotating through 360° as the mechanical system makes one revolution around the Sun.

The Fortran source code used to generate the data used in this analysis is at the end of this paper.

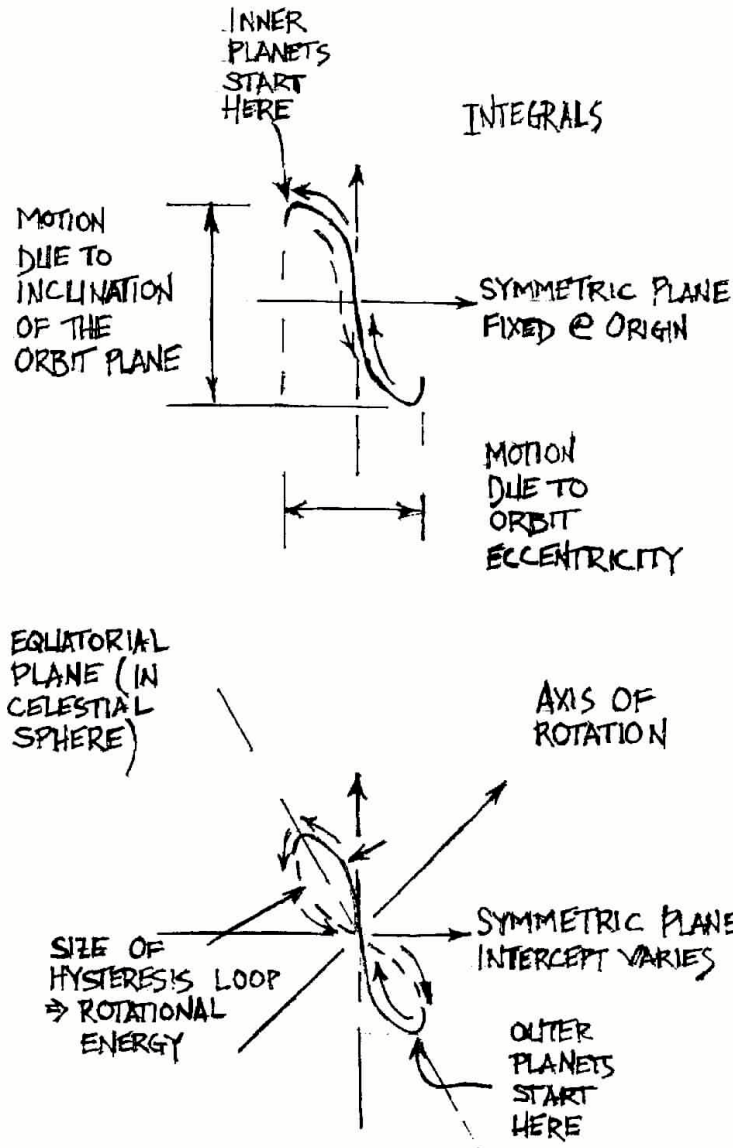


Earth and its Orbit around the Sun

Analysis

Matrices allow points and vectors to be rotated in space about coordinate axes, translated in space, and referenced relative to other reference frames. The above describes a transformation to a plane at some angle to the ecliptic, and this transformation can be done

by a matrix once the plane itself is found. This allows us to look at the motion of the planets in a rotating coordinate system fixed upon any given planet. The transformation is linear and its eigenvectors are presumably the principal axes and the eigenvalues are perhaps the relative strength of these axes.

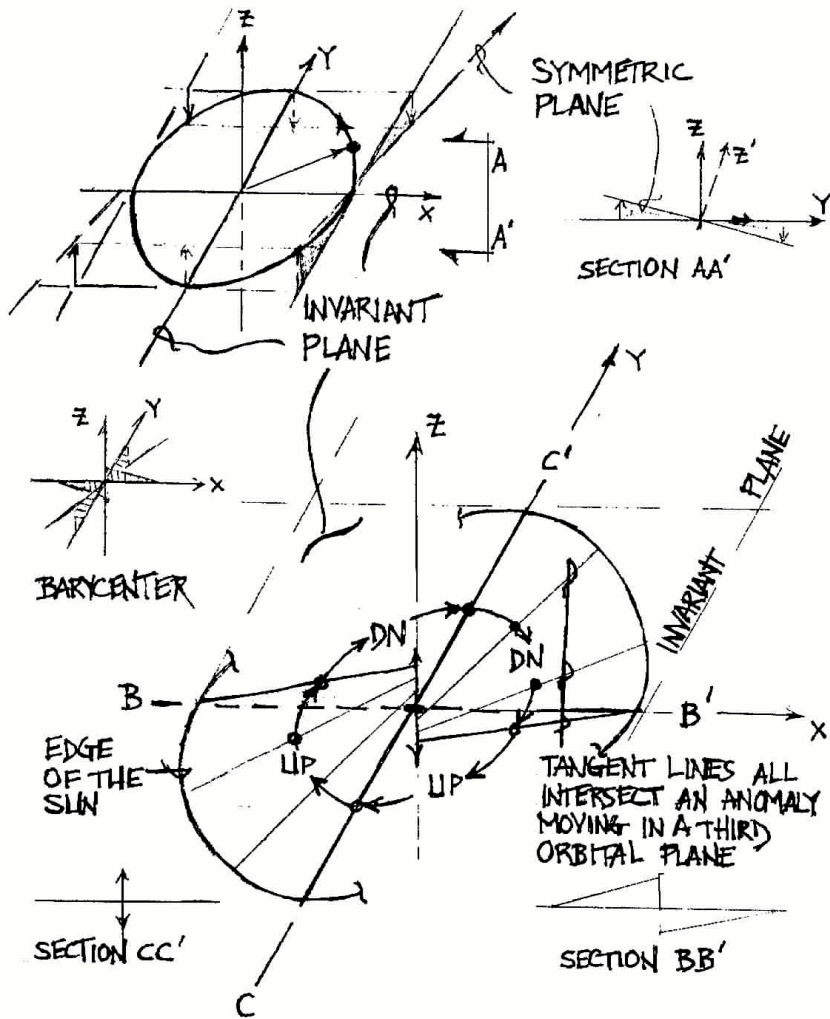


Planetary Motion versus the Symmetric Plane

Getting a little ahead of the game, consider the possibility that 3D space in the region of a star is built just like the system wave (3), then the approximation to the ellipse in (1) is

exact with just one term, and for all ellipses - not just those with small eccentricity. This greatly simplifies all calculations if they are performed in this hyperspace construct.

The following figure shows some geometry of this hyper plane.



The system wave theory (3) is based on the premise that the electromagnetic nature of Earth (and the other planets) acts to align the planet's axis of rotation so that it always points to the same point in the celestial sphere. The planet's equator is always aligned along the curved s-shaped fragments (which are not to scale, and the symmetric plane is skewed to

the celestial equator - these two properties are elaborated in the next paper), shown all together in the above illustration.

Individually, each planet's sine wave fragment is traced out two times in the course of an orbit around the sun, following the same path forward and backward. Plotting this same path versus the symmetric plane with the origin fixed at the sun (without the y-intercept varying) results in a closed figure-8 loop. In this regard, each planet is in a stable orbit versus an equilibrium point. These equilibrium points are strange or chaotic attractors (14), mathematically. Celestial mechanics considers them halo orbits around stability points, in the rotating coordinate system (taking out the bias in the x-coordinate generates an ellipse versus the symmetric plane).

You will notice that the computer algorithm is itself an approximation, used to find the heliocentric coordinates in an "inverse problem" type of discovery here. It uses a simple sine variation about the semi major axis (the first term approximation for the small eccentricity ellipse (1) model), which gives the kind of Keplerian approximation of planetary positions accurate to what an amateur astronomer might be able to achieve. This paper seeks to solve Kepler's problem with a slightly more accurate set of data, and (1) shows these data to be accurate to four to six significant digits, which is to the accuracy of the data input.

```

C
C The Inverse Problem of Celestial Mechanics
C by WH Clark
C
PROGRAM SEROS
implicit REAL*8 (a-h,o-z)
REAL*8 Elemts(5,9),Hx(40),Celestex(10,40),Celestey(10,40), LSfit(40,2),Bary(10,40) OPEN
(6,FILE='a.out')
1 FORMAT (1X,2E8.4)
pi = DACOS(-1.D0)
C
C Read orbital elements into an array for the nine planets; source:
C Celestial Basic: Astronomy On Your Computer
C by Eric Burgess, Fellow Royal Astronomical Society
C (1960 Epoch) published by Sybex
C
DATA Elemts / 0.387100,0.079740,2.735140,0.122173,0.836103,
+ 0.723300,0.005060,3.850170,0.593410,1.331680,
+ 1.000000,0.017000,3.339290,0.000000,0.000000,
+ 1.523700,0.141704,1.046560,0.031420,0.858702,
+ 5.202800,0.249374,1.761880,0.019720,1.745330,
+ 9.538500,0.534156,3.125700,0.043633,1.977458,
+ 19.182000,0.901554,4.490840,0.013960,1.288050,
+ 30.060000,0.270540,2.334980,0.031416,2.291620,

```

```

+ 39.440000,9.860000,5.231140,3.001970,1.918120 /
C
C Read data into heliocentric longitude array in 10 degree increments
C where Hx(1) = 0 degrees; Hx(36) = 350 degrees
C (calculation units are in radians)
C
Hstep = 0.D0
DO 10 n=1,36
Hx(n) = Hstep
Hstep = Hstep + pi/18.D0
10 END DO
C
C Calculate in heliocentric coordinate system at each Hx value::
C x-coordinate, Celestex(planet,Hx) = distance from sun
C y-coordinate, Celestey(planet,Hx) = distance from ecliptic
C
DO 30 m = 1,9
C write (6,*) m
DO 20 n=1,36
Celestex(m,n) = Elemts(1,m) + Elemts(2,m)*
+ DSIN(Hx(n) - Elemts(3,m))
Celestey(m,n) = Elemts(4,m)*
+ DSIN(Hx(n) - Elemts(5,m))
C write (6,*) Celestex(m,n), Celestey(m,n)
20 END DO
30 END DO
C
C Calculate the simple (linear) least squares fit at each Hx value
C LSfit(Hx,1) = slope and LSfit(Hx,2) = intercept
C Correlation ?
C
DO 50 n = 1,36
sumx = 0.D0
sumy = 0.D0
sumxx = 0.D0
sumyy = 0.D0
sumxy = 0.D0
DO 40 m = 1,9
sumx = sumx + Celestex(m,n)
sumy = sumy + Celestey(m,n)
sumxx = sumxx + Celestex(m,n)*Celestex(m,n)
sumyy = sumyy + Celestey(m,n)*Celestey(m,n)
sumxy = sumxy + Celestex(m,n)*Celestey(m,n)
40 END DO
LSfit(n,1) = (9.D0*sumxy - sumx*sumy)/
+ (9.D0*sumxx - sumx*sumx)
LSfit(n,2) = sumy/9.D0 - LSfit(n,1)*(sumx/9.D0)
write (6,*) LSfit(n,1), LSfit(n,2)
50 END DO
C
C Determine the position of the planets relative to these 36 lines
C (the x-distance from the sun is radial, so it is unchanged)
C

```

```
DO 70 n = 1,36
DO 60 m = 1,9
Bary(m,n) = Celestey(m,n) -
+ LSfit(n,1)*Celestex(m,n) + LSfit(n,2)
60 END DO
70 END DO
C
C The coordinates of each planet versus the Invariant Plane are
C (where n = heliocentric longitude in degrees) n = 1,36
C Celestex(planet,n) ; Bary(planet,n)
C Read these to an external file so they can be studied in Excel
C
DO 90 m = 1,9
write (6,*) m
DO 80 n = 1,36
C WRITE (6,*) Celestex(m,n), Bary(m,n)
80 END DO
90 END DO
END PROGRAM
```

NOTE: The second Earth value in the Read table was 1.01700 for the plots; this may change the graphs slightly.

3 The System Wave

ABSTRACT

The motion of the planets is transformed into a new coordinate system, i.e. with respect to the invariant plane and a model is constructed to explain the behavior of the planets, collectively. The result is a physical graph of two distinct wave patterns, with the Earth (or moon, depending on the orientation) at the "g-spot." One pattern applies to the inner planets, the other to the outer planets. This model fits several aspects of the planets: inclination of axis of rotation, ellipticity of orbit, and inclination of the orbit. These parameters are not explained by any other theory. The solution is an envelope curve (27), or an envelope surface in the non rotating system.

A Geometric Study

The first objective of the analysis is to create a more stable system by better explaining systematic deviations from the symmetric plane. In order to further define the motions, first introduce another variable: the planet's axis of rotation. This axis is known to always point to the same spot in the celestial sphere, throughout its revolution around the sun. During this movement, the planet moves evenly above and below the ecliptic plane. These characteristics can be conveniently represented if the planet is considered to be associated with a longitudinal wave. If the axis of rotation is always tied to this sinusoidal wave, then as the planet moves about the sun the elliptical shape of the orbit causes the planet to move closer and farther away from the sun. If this motion happens on a sinusoidal wave, the axis will always point in the same direction.

Otherwise, the choice of a sinusoidal waveform explains the three movements that occur simultaneously with respect to the new system plane: the elliptical shape of the orbit, the movement above and below the system plane, and the axial inclination of the planets pointing to a fixed spot in the heliocentric sphere while the other motions occur. The data used in the analysis is listed in Table 1. The semimajor axis is an approximation of the planets' average orbital distance from the sun. A value "X" is the range of motion of the planet in its orbital plane.

Planet	X	Axial inclination	Semi major axis
Mercury	5.8 E7	0.0 degrees	1.2 E7 km
Venus	1.1 E8	R179	6.7 E6
Earth	1.5 E8	23.5	2.5 E6
Mars	2.3 E8	25.2	2.1 E7
Jupiter	7.8 E8	3.2	3.8 E7
Saturn	1.4 E9	26.8	7.8 E7
Uranus	2.9 E9	R98	13.2 E7
Neptune	4.5 E9	29.0	3.7 E7
Pluto	5.9 E9	90.0	1.5 E9

Table 1

The next task is to find the wavelength of a common "system wave." After trying several solutions, it was evident that the outer planets would be the controlling elements, so the search focused on the lowest common denominator of their orbital range. This value, 23.3 E6 km also had to fit the entire range of movement of the inter planets.

The second column of Table 2 lists the aspect of each planet upon this wave – the remainder of division by the proposed wavelength. This value was used to situate each planet at the proper location upon the wave. Note that positions are only approximate in this two dimensional system: the wave is actually a spiral or helix in three dimensions.

A possible presentation of the "system wave" is a family of curves called the Cochoids of Nichomedes, with the parametric equations:

$$x = a + \cos t \text{ AND } y = a \tan t + \sin t$$

These generate a two-part curve: a loop similar to a prolate cycloid, and a shell. This solution will have some unique possibilities in the analysis of orbits.

The second objective of the analysis was to further develop the study to define the uniform deviations from the new system plane, so that a more stable system could be devised. This has been done for the original positions by simply introducing another variable and trying to match it. By seeking a close fit to this third variable, axial inclination, the affect was to more closely resolve the previously analyzed parameters. This affect becomes more pronounced as the analysis continues in greater detail.

<i>Planet</i>	<i>Division by 23.3 E6</i>	<i>Inclination of orbit to the ecliptic</i>	<i>Maximum distance from the ecliptic</i>
Mercury	2 + 1.1 E7 km	7.0 deg	7.0 E6 km
Venus	4 + 1.7 E7	3.4	6.5 E6
Earth	6 + 1.0 E7	0.0	- -
Mars	9 + 2.0 E7	1.9	7.6 E6
<i>Apollo Group</i>	9 + 7.8 E6	-	-
<i>Belt Asteroids</i>	16 + 2.2 E6	-	-
<i>Trojan Group</i>	33 + 3.6 E6	-	-
Jupiter	33 + 1.1 E6	1.3	17.0 E6
Saturn	60 + 2.0 E6	2.5	61.0 E6
Uranus	124 + 1.0 E7	0.7	35.0 E6
Neptune	193 + 3.1 E6	1.8	141 E6
Pluto	253 + 5.1 E6	17	172 E6

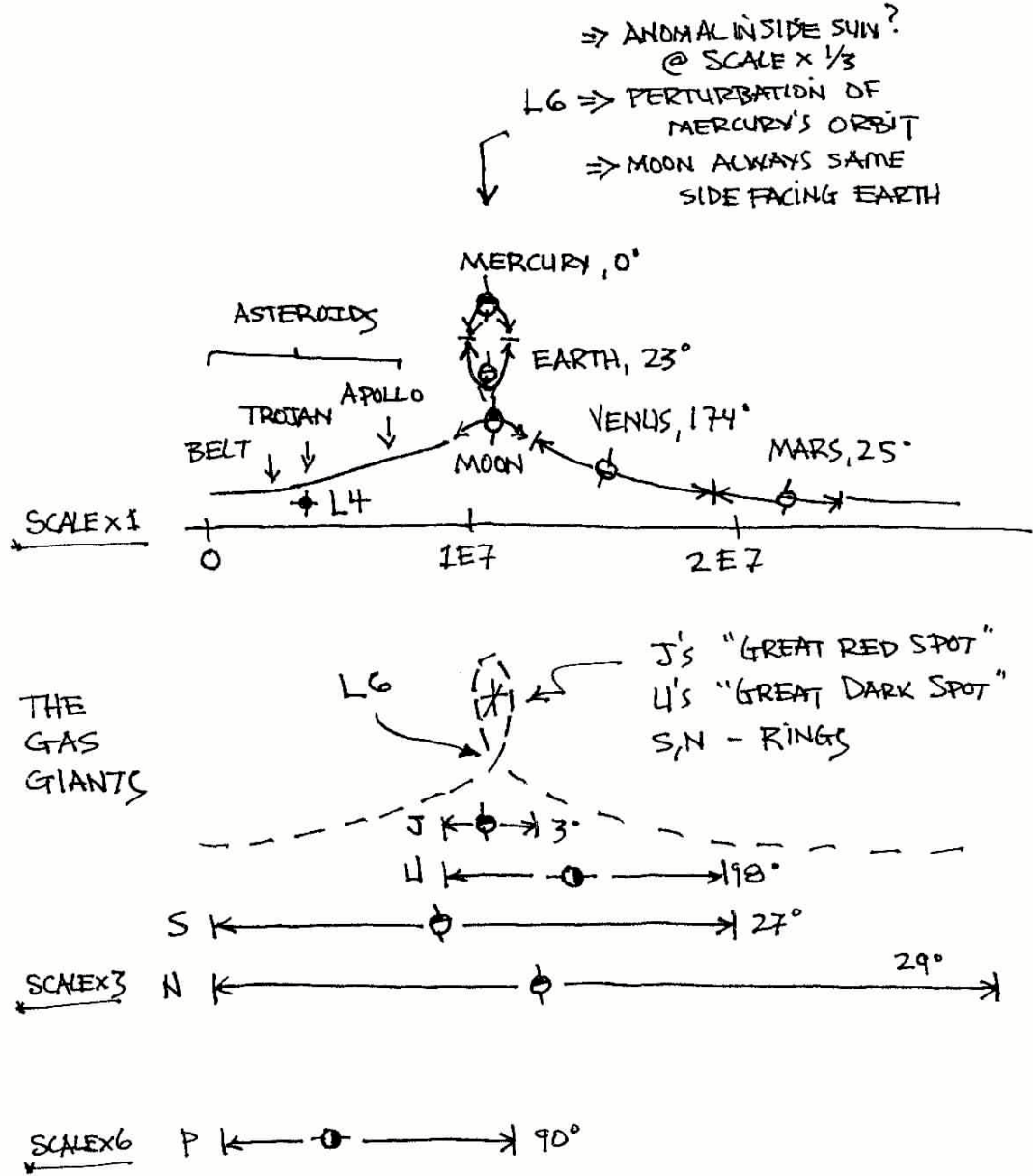
Table 2

Later is a sketch of the planets situated upon either of the two parts of the cycloid, according to the average orbital distances in Table 1. The axis is shown as an arrow pointing according to the right hand rule for the planets' rotation. The positions of the various planets on the waveforms can now be fit to the wave.

First, the motion of Mars moves uniformly on either side of an inflection point in the wave, its axis inclined at about the right amount. Jupiter does the same thing on the lower wave, which arches less to correspond with the lesser axial inclination of Jupiter's axis. Uranus, also on the lower wave, moves through an entire wavelength, with an eight degree axial inclination off the vertical, slightly greater than Jupiter – since it traverses more of the wave.

Next, Saturn on the lower wave traverses two wavelengths, with a much greater axial inclination. The center point of the wave can be thought of as an inflection point, motivating the body to rotate in order to maintain rotational stability – a two dimensional vista of the singularity at the sun discussed earlier. It is interesting to note that all four planets (with the possible exception of Pluto) have almost the same rotational period – about ten hours. This

would imply some similarity in the wave analogy, and it is there in the form of the inflection of the wave traveled.



The System Wave

Back to the upper waveform, there is Mercury shown atop the loop, It moves back and forth on the waveform the distance of its ellipticity, maintaining its axis pointed in the same direction. Although the planet is known to have an axial inclination of zero, the wave may be inflected so that the pattern traced is less exaggerated. The geometry of that area of the plane fits this expectation.

The Earth-Moon system is the most striking aspect of the system, buttoning up the sides of the loop. They are unique in the solar system, the Moon being the largest satellite with respect to its host in the whole solar system. This may be a necessary prerequisite for matter to exist in dynamic stability at the cusp of the wave front as shown. In which case Earth occupies the upper portion of the loop, the moon the lower, in its own orbit. The moon's orbital plane is inclined at five degrees to the Ecliptic, as is indicated by the inflection of the lines (versus the much higher inflection of the upper cusp, indicative of Earth's 23° axial inclination).

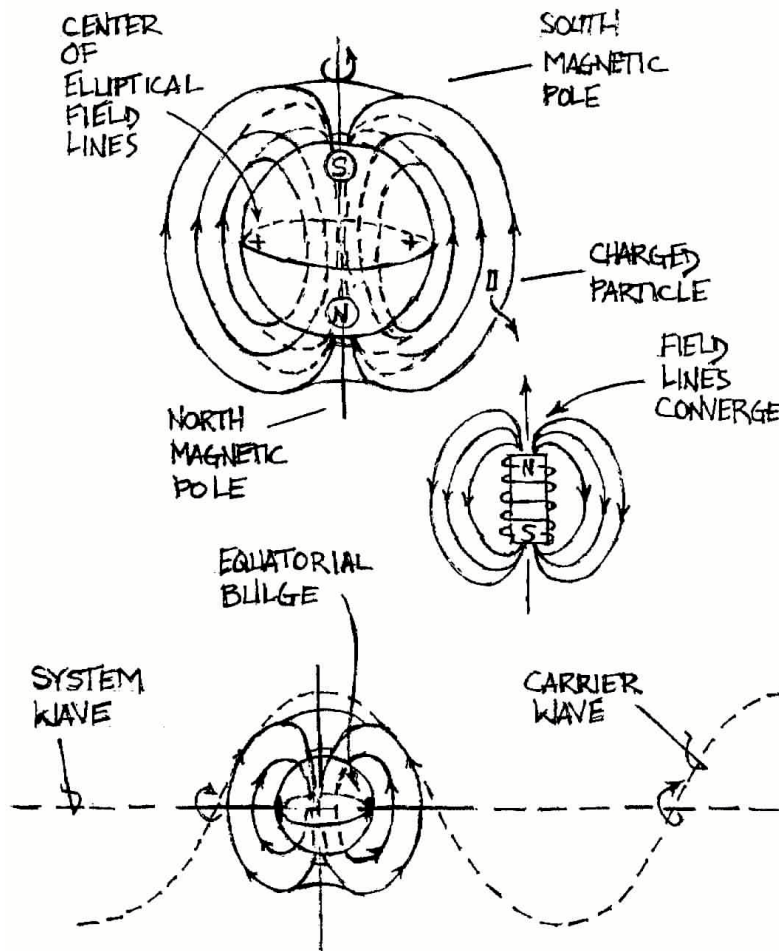
The close symmetry of the least squares fit to a sinusoidal slope and a sinusoidal motion of the y-intercept, and the uniform displacement of the midline of the sinusoidal motion of the planets about this plane, then a two-part pattern such as that envisioned most closely realizes both conclusions. In affect, the defining plane of motion then becomes the bottom half of the above parametric equations wave for the inner planets (see Table 3).

Planet	Movement w.r.t. the system plane (Au)	Center line	Radius
Mercury	$-0.30 + 0.21 = 0.51$	-0.05	0.26
Venus	$-0.27 + 0.17 = 0.44$	-0.05	0.22
Earth	$-0.19 + 0.17 = 0.26$	-0.06	0.13
Mars	$-0.19 + 0.09 = 0.28$	-0.05	0.14
Jupiter	$-0.10 + 0.02 = 0.12$	-0.04	0.06
Saturn	$-0.30 + 0.17 = 0.47$	-0.06	0.23
Uranus	$-0.77 + 0.74 = 1.51$	-0.03	0.75
Neptune	$-1.21 + 1.21 = 2.42$	0.00	1.21
Pluto	$-1.10 + 1.60 = 1.70$	-0.25	0.85

Table 3

Another way to look at the "system wave" is as an envelope curve (27) or "envelope surface" made up of segments of all nine planets' orbital configurations. Thus the analysis here is more than a geometric curiosity but a common aspect of complex dynamical systems. It is also important to note that, if nothing else, the "system wave" shows an exact correlation between orbit eccentricity and inclination or the orbit (also, perhaps, inclination of the axis of rotation) as was surmised in modeling the low eccentricity ellipse (2) , and vice-versa. Presumably perturbations to the 3BP model by other planets would cause rotation on the axis, as was suggested here.

Note that the origin of the "system wave" is at the sun, not at barycenter. That explains why the plot of the orbits versus the symmetric plane is skewed ~ sinusoidal, and not elliptical. (2)

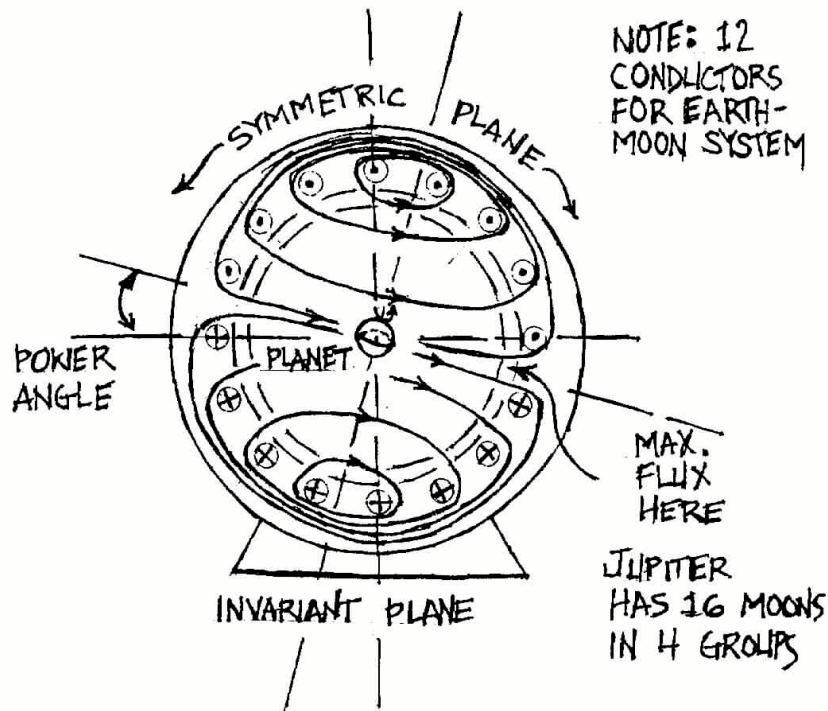


An Electromagnetic System Wave versus the Magnetic field of a planet

Analysis

Continuing the electromagnetic analogy of (2), this helps to conceptualize the forces interacting in determining the motion of the planets - not just their orbital elements, but also the inclination of their axis of rotation. The latter comes into play because of the magnetic core of the planets. The inclusion of axial inclination adds an aspect of electromagnetism (EM) to the analysis by assuming gravity has an EM carrier wave intrinsically associated with it. (21)

The illustration shows how a planet's magnetosphere might interact with an external EM force field, like a motor/generator combination. (29) Notice the ellipses of field lines are all centered at the same location, near the equatorial bulge, like a $1/r$ force field. A motor (generator) has a rotor and a stator. Consider the fixed, unmoving part of the device, the stator and its poles .

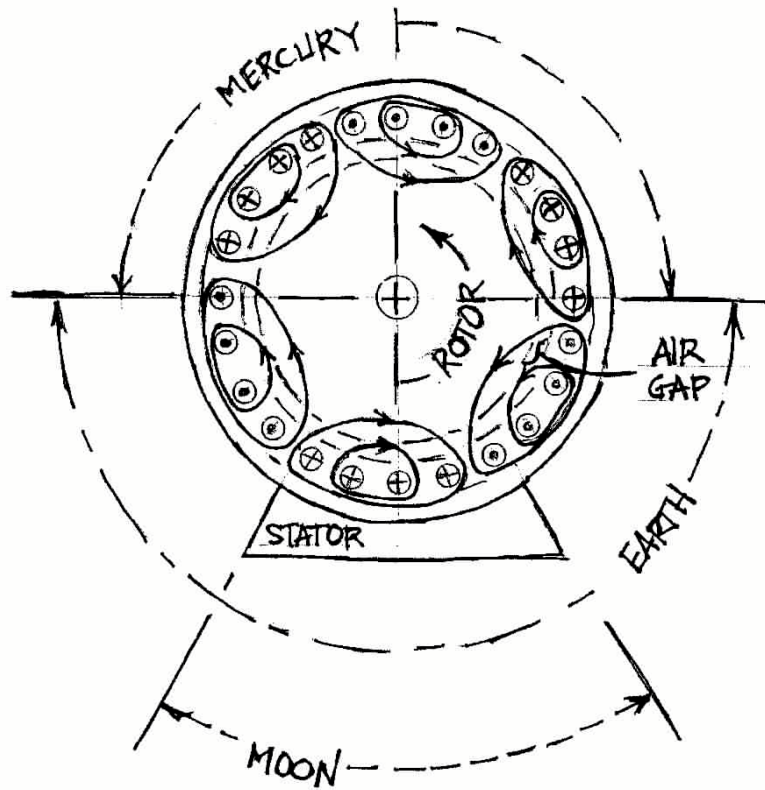


Electromagnetic Linkage between Planet and System Wave

The Earth, as already noted, is the stator. The rotor is formed between the symmetric and invariant planes which are both fixed - at least in the planet centered reference frame.

Notice the two planes, the symmetric and invariant planes, are the north and south poles of a single EM field, fixing the position of the planet between them. The two planes are both hinged at the same origin but the symmetric plane has an off center hinge and this makes it possible for the two planes to hold all the planets between them.

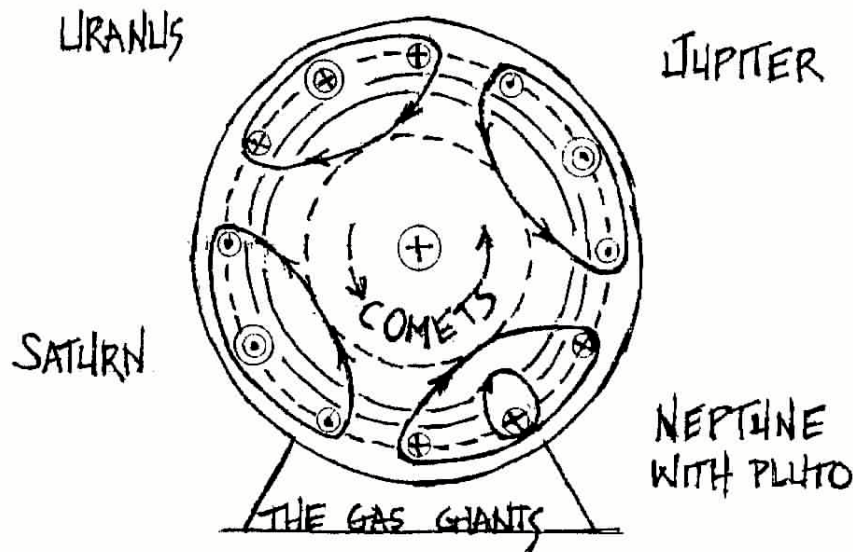
The loop in the system wave can, in this analogy, be thought of as a six pole stator.



Electromagnetic Analogy to the System Wave Loop

The overall action of this loop, then, is as another Lagrange Point with rotational properties of force at a distance, just as exhibited by the $1/r$ EM phenomena. Overall, the system acts as a three phase electrical system (3BP) with Mercury rotating to face the anomaly just like the Earth's moon rotates to always show the same face to Earth.

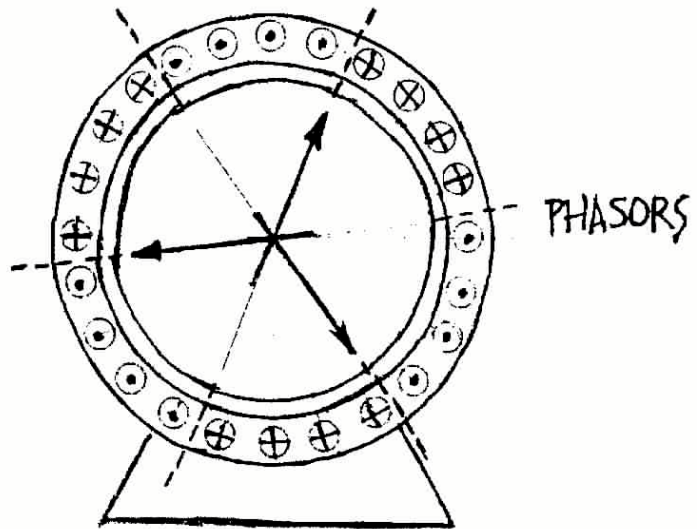
The outer planets have a different composition than the inner planets. They are large gas giants, versus the solid matter inner planets. So it is logical to present loops in the outer planets as less intense, more like a four pole stator.



Electromagnetic Model of the Outer Planets

Here one pole is associated with each of the gas giants, forming rings or a "Great Red Spot" (Jupiter) or "Great Dark Spot" (Uranus) as the case may be. Pluto is shown associated with Neptune because Pluto passes inside Neptune's orbit at times. This is not to imply that Pluto is part of the gas giant model, but to suggest how the next fractal level is associated with the outer planet level. Perhaps at Pluto the system behaves only like a simple two pole apparatus, the invariant vs. symmetric planes only, being too weak to force any more complex motion; thereafter dissipating and leaving only the symmetric plane, until approaching another star system.

It has already been noted that a halo orbit in a rotating coordinate system about a Lagrange Point is the same as each planet's motion versus the symmetric plane. Thus, the transformation of coordinates to the symmetric plane establishes a specific Lagrange Point for each planet. (2) In this way, the loop acts as an attractor (14). The following illustration shows the rotating flux in the frequency domain for a three phase electrical system.



The Three Phase System

It may be convenient, in applying some of these abstract notions to the study of electrical motors and generators, that the Three Body Problem is shown by analogy as a Three Phase electrical system. Perhaps this will help to reduce the harmful harmonics created by rotating electrical devices and make them more efficient.

4 A 4BP with a Binary Planet

It has been shown (1) that an ellipse of small eccentricity (i.e. .01 or less) can be constructed simply as:

$$r = 1 - \frac{e^2}{1!} \cos\left(f + \frac{\pi}{2}\right) - \frac{e^3}{2!} \cos\left(2f + \frac{\pi}{2}\right) + O(e^4)$$

Consider a 3BP set up so that the second primary mass (the smaller mass) is a binary planet - a small mass orbiting very close to a much larger mass. If the second mass makes one orbit per revolution (all three primary bodies are in the same orbital plane), this simulates the second term of the equation and the Center of Mass (CM) of the binary planet will move in an ellipse to $O(e^3)$. Or, if the larger binary mass is in an elliptical orbit, the action of the second mass of the binary makes the CM move in a circular orbit, to the same accuracy. These neglect the affect of the smaller planet on the barycenter, which parallels the second term of the equation (5); and the slight variation on mean motion, which is in any event a secular variation that averages to zero for each orbit.

Introducing a fourth mass orbiting the larger binary at 2x per revolution can make the fit to the CM orbit exact to $O(e^4)$ so that the entire system behaves as a large primary near barycenter and another CM in circular orbit around it. The same stable orbit could be simulated by a small mass orbiting the primary at barycenter 2x/rev, e.g. as an anomaly within the sun compensating for motion.

A more likely scenario would have the smaller planet(s) out of plane, all together simulating an orbit of CM in a plane at a small inclination to the original orbital plane, where the driving influence toward stability is a constant direction of the angular momentum vector, the CM remaining in a fixed orbital plane.

In so doing, placing the small fourth mass near the best orbit would drive it into that best orbit, making the fourth body exist in a stable Lagrange type orbit. Thus a new Lagrange point, or stable point, for the 4BP - actually two such points, as another is at barycenter. Additional stable points happen at regular intervals as the nominal eccentricity of the large primary binary planet increases (ultimately building up a "system wave" (3) in a complex natural system). That is, the 4BP is first stabilized as indicated, upon a circular orbit (or consistent orbital plane, as the case may be); then a fifth body is added to compensate for a further increase in eccentricity/inclination - each stage with associated Lagrange points. This

follows the method of consecutive CM calculations via a "chain of barycenters" technique, proven in the theory of Celestial Mechanics as a means of computing CM values for many body problems.

With each iteration the large primary mass is in an increasingly elliptical orbit (or out of plane orbit), whereas the small system remains in the same orbital plane, e.g. it's center of mass.

Eventually, as more terms are considered in the equation, the collection of masses becomes more like a single gaseous planetary entity where the gravitational field is comprised of several anomalies - e.g. Jupiter. This analysis has built up a Fourier Series representation of the orbit to match the Fourier Series way that a model of a planet's gravitational field is developed via satellite geodesy.

The solar system has an invariant plane, consisting of the angular momentum vector/plane. Let the primary body be Jupiter with the sun at barycenter. Now each planet is driven to a stable orbit, by mass distribution (including anomalies within the sun itself) to move in the plane perpendicular to the system angular momentum vector, i.e. the invariant plane.

This type of discrete mass distribution would presumably make it difficult for a large planet sized mass to reside at the sun-Jupiter equilibrium point. Hence, the asteroids that orbit the L4 and L5 points, but nothing exists at the Lagrange points themselves.

5 A Locutus at Barycenter

This report attempts to establish a physical phenomena to resolve the "error" in the second term of the trigonometric series representation of the ellipse (1). As illustrated, the deviation of second order is a cosine function of twice the frequency and a fraction of the amplitude as the first term. The two perturbations together produce a result accurate to at least eight significant digits (i.e. for $e=.01$, the maximum permissible). Given all the other unknowns in a system of many bodies like the solar system, this is the best approximation one should reasonably expect since anything more accurate would be inconsistent with the situation and the natural order of things.

Imagine a simple 3BP consisting of an object in Earth orbit. Since 99.5 % of all satellites are in nearly circular orbits, $e < .01$, it is reasonable to suppose the alleged sinusoidal variation from the unit circle orbit is bona fide.

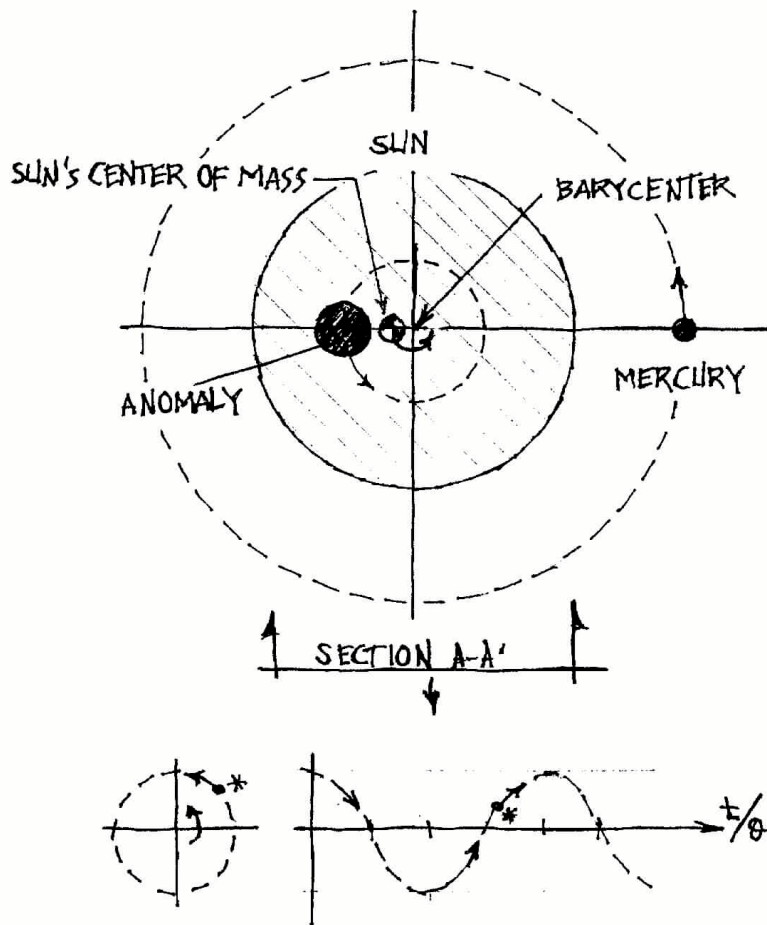
The error in the ellipse has the same pattern and frequency as the motion of the barycenter in a sun/planet/satellite 3BP. That is, if the planet were in a low eccentricity orbit, this orbit could be modeled accurate to eight significant digits by simply ignoring the motion at barycenter - i.e. fixing the distance from the origin for larger primary body near the sun, and varying the motion of the planet by just the first term of the series representation of the low e orbit. (1)

Another solution to this 3BP is an anomaly within the sun, in the same plane as the planet's orbit, moving in such a way as to result in a 3BP solution to the circular coplanar problem.

A combination of these two alternatives would explain the evolution of a stable planet. An unstable circular orbit initially creates a virtual singularity in the sun, around which an anomaly develops over time (like a pearl around a grain of sand in an oyster) as the planet's orbit becomes stable and elliptical.

This kind of scheme is suggested by the "vortex" at the origin of the symmetric plane (2). Any new body or cloud of mass entering the solar system would assume a circular orbit initially, a very dangerous even for the dynamically delicate solar system (circular orbits being a singularity). Instead of initiating a point singularity within the sun itself (a uniform hoop of mass is a point gravity source) - a disastrous calamity - the asymmetric origin of the symmetric plane simulates a non-circular orbit, until such time as overall equilibrium can result for the solar system as a whole in the presence of this new massive body.

At the same time, this symmetric plane makes the solar system seem a point source from a long distance (e.g. a "black hole"), thus allowing it to attract mass and energy sufficient to make it a self sustaining system. Conversely, as the star weakens the invariant plane weakens, and the symmetric plane strengthens until it reaches the point where the vortex overcomes the point singularity inclination and the system explodes and then the process starts all over again, the anomaly in the sun forming the core for the brand new star. With each iteration, the solar system becomes more and more complex (ours is likely among the very oldest star systems - with only one gas giant remaining), until eventually the star can possibly switch over without altering gravity, leaving one or two planets unchanged: Earth is like that.



Precession of Mercury's Perihelion

NOTE: By fractal theory, if the L6 point causes anomalies in gas giants - Jupiter's Red Spot and Uranus' Dark Spot - then it should also cause anomaly within the sun as posed here.

6 Gravity Wave Interference

ABSTRACT

Gravity is a wave; that much is accepted by all scientists. This short paper shows how gravity, as a wave, acts as a perturbing force to cause the elliptical shape of the orbits of planets around our sun. That is, as the constructive interference between sinusoidal waves. This report will show the efficacy of the transformation of coordinates to the symmetric plane (2), in that orbital motion of natural systems of many bodies such as the solar system requires one less parameter than in a generalized 3D inertial reference frame. This is a strong validation of the symmetric plane concept, and the elegant mathematical development to follow suggests that this construct may be fundamental to other many body dynamical systems.

It has previously been shown that a symmetric hyperplane can be found for the solar system by which the motion of the nine planets is completely symmetrical. (2) That is, a body in an elliptical orbit exhibits sinusoidal behavior in two respects: (i) in the plane of the orbit (this causes the ellipticity), and (ii) perpendicular to the plane of the orbit (this causes the inclination of the orbit with respect to the symmetric plane). This is simply an application of the well known principle of analytical geometry by which it can be shown that an ellipse is the combination of two sinusoidal wave forms.

The exact position of a planet with respect to this symmetric plane (henceforth the analysis will pertain to a single satellite orbiting a central body) can therefore be represented by the coordinates (in a rotating coordinate system):

$$x = a + b \cdot \sin(\phi + c)$$

$$z = d \cdot \sin(\phi + e)$$

where ϕ is an angle from some reference point, x is the radial distance from the central body, z is the perpendicular displacement relative to the symmetric plane, c and e are phase angles.

The first equation for x is the radius vector, and this equations corresponds to the well known expression

$$r = a - ae \cdot \cos E$$

where E is the eccentric anomaly (one could just as easily use \cos in the expressions for x and z ; the phase angles c and e make it possible to use either \cos or \sin). The eccentric anomaly is very nearly the same as the angle ϕ for orbits of small eccentricity.

Observe that this solution attempts to use a Fourier Series solution to the orbit - a solution practiced 100+ years ago by Poincare et al but discarded because it implied an unstable dynamic system - thus the term we have here is the first term in a Fourier Series, and need only be approximate because other terms will be added for the exact solution. This should not be hard to conceptualize. All orbital mechanics assumes gravity of bodies acts as a single point mass. All this series solution implies is that the body is not of uniform density, and that different aspects of density act individually upon an object in orbit, summing to the actual orbit, e.g. satellite geodesy.

The correspondence to the normal orbital elements is easily visualized:

eccentricity is the ratio of b/a

inclination is the ratio of d/a

argument of periapse is the angle c

longitude of ascending node is the angle e

eccentric anomaly corresponds to ϕ

semi major axis corresponds to a

Variations in the orbital elements is an important part of orbital mechanics. Unlike in the conventional coordinate system, these are not abstract concepts. They are rather the result of combining sinusoidal wave forms, as in the interference patterns between gravitational waves. Any shape or orientation of an orbit can be made by the combination of sinusoidal waves. That is the basis of Fourier Analysis. The gravitational field of the Earth is modeled, in fact, as terms in a Fourier Series; making it a straightforward process to correlate aspects of the gravitational model to their precise affect upon the orbit of a satellite.

(1) a sine wave in phase with x changes eccentricity

(2) a sine wave in phase with z changes inclination

(3) a sine wave in phase with c causes precession of the periapse

(4) a sine wave in phase with e causes the ascending node to precess

The coordinate system thus rationalizes the actual force of gravity, and its affect upon physical bodies by virtue of a constructive interference between waves.

Recall that in the graphical derivation of the symmetric plane, (2) the phase angles for many of the planets were the same. This reduces the number of unknowns in the analysis of a many body problem. The results of that paper - showing all the planets with prograde rotation in phase; all the planets with retrograde rotation in phase; and the two sets of planets

π radians out of phase - also is a proof of this hypothesis, as it shows that all nine planets can be represented by the set of equations as stated.

Be that as it may, eliminating unknowns is very important in celestial mechanics. The derivation of the Jacobi Integral in the early days of celestial mechanics resulted in literally thousands of new papers in a field that had been in the doldrums. The above coordinate transformation makes it possible to represent a body in orbit using just five variables; whereas all existing systems must use six. This is a significant development. (The transformation of coordinates into the symmetric plane eliminates the phase angle for x from both equations if all planets are in prograde rotation or all are in retrograde rotation.)

A common way to reduce the number of variables in problems involving three or more bodies is to assume all the bodies move in the same plane. When studying such a problem in the new coordinate system, two variables are eliminated (d and e); this means that a coplanar 4BP in this coordinate system has only 12 unknowns - the same number of unknowns as the coplanar 3BP in conventional coordinates. This means it is mathematically possible to extend our understanding of the 4BP from a handful of papers into many thousands of papers with relative ease. Also, notice that analytical method creates a system of variables which are a function of sines and cosines only. Thus, all higher order derivatives exist when they are used in the differential equations of motion, which is convenient.

Anti Gravity

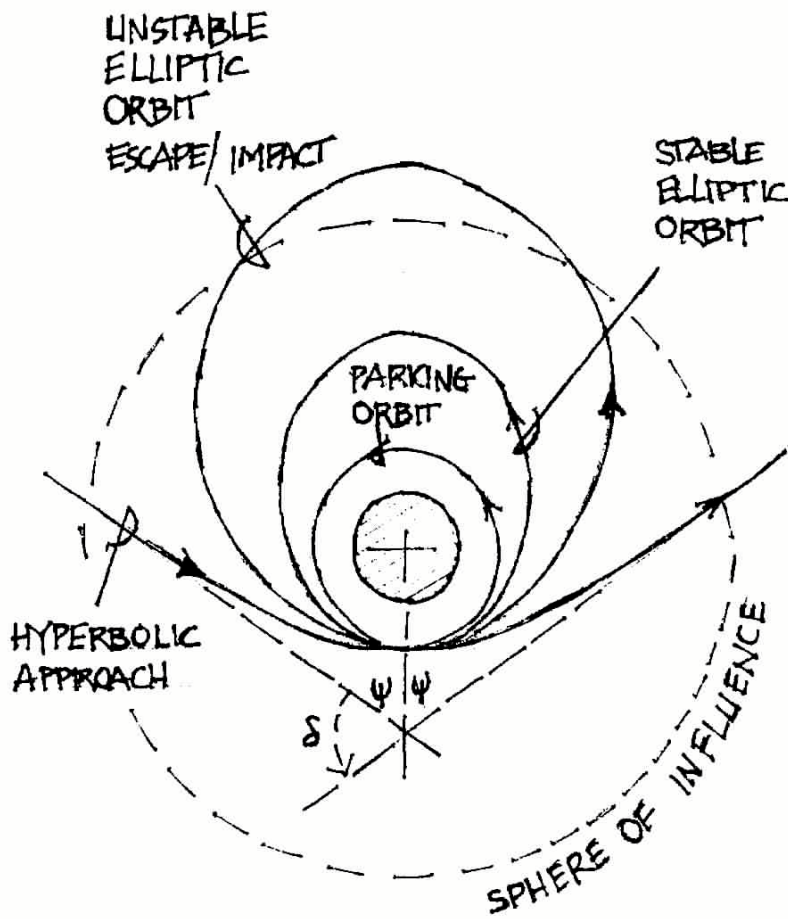
You will recall the fact that at some level the mass of each planet in the solar system can be approximated as a point mass (INTRO). If the body were completely spherical and of uniform density, it would act like a point mass any where outside the sphere. The more inconsistent and asymmetrical the body, the farther away one must go before it acts like a point mass. It's conceivable that this demarcation in the case of satellites orbiting Earth is the emergence of relativistic components to the gravity model of Earth.

It is easy to show, using simple geometry (Moulton) that a spherical shell of uniform thickness and density behaves just like all the mass were concentrated at the center. Inside this sphere, all the forces cancel so an object there experiences no gravity at all. Inside the thin crust, the Earth can be closely approximated by concentric shells of nearly uniform density, each of which collapses to a point mass at the center. This much is evidenced by the existence of relativistic phenomena at relatively low Earth orbits.

For the purpose of argument, assume that each of these shells has a unique harmonic frequency, and it emanates a gravity wave - perhaps perpendicular to the N/S

magnetic pole, similar to a 3D electromagnetic wave. (29) The point at which these waves begin to combine constructively is the event horizon where relativistic phenomena can be detected in perturbations of orbiting satellites.

Assuming such large scale wave fronts exist in fact (this would be a very efficient way for a planet to exert its gravitational influence/connection with distant objects, and nature is the penultimate when it comes to conserving energy) then it should be possible to arrange a destructive interference of one or more of these shells, maybe enough to make applications at large power plants cost effective.



II. Optimizing Interplanetary Trajectories

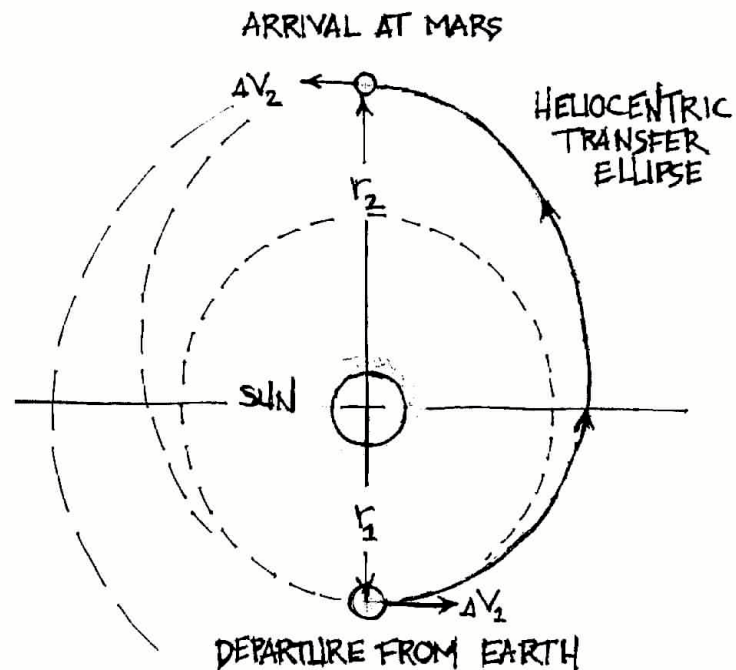
ABSTRACT

Interplanetary trajectories are typically too nonlinear to be modeled directly, much less optimized, easily. The problem is so nonlinear, in fact, that none of the proven optimization algorithms work, and a totally random scheme called genetic programming must be used. The purpose of this report is to show that an interplanetary trajectory can in fact be solved, and optimized, without resorting to any optimization methods at all. The method has been proven valid on a computer simulation of the Earth to Mars trajectory, which finds a more efficient mission profile than any other optimization technique. This is done by simply modeling the algorithm on some of the fundamental rules of orbital mechanics. This is not only an effective way to structure an algorithm but provides many more insights into the dynamics of the problem than statistical/optimization or random/genetic schemes.

Trajectory Correction Maneuvers

It is generally accepted that the most efficient way to apply a thrust to a spacecraft is along the current velocity vector. Doing so applies 100% of the fuel energy to increasing velocity. Thus, if you want an efficient trajectory, you plan on applying all thrusts along the respective velocity vectors. The art is to design the flight path so that the resulting changes in the orbital path alter the architecture of the orbit in a way that is advantageous to the overall mission objectives.

An in-line thrust alters the orbit in different ways, depending on where it is applied. Consider a Hohmann Transfer from Earth to Mars, in which the objective is to find a neighboring trajectory that reaches Mars faster and with less fuel. An extra thrust at periapse increases the eccentricity and the semimajor axis of the orbit (unless otherwise noted, all thrusts are forward to speed up and not in reverse to slow down), the reverse action of aerobraking. This increases the distance traveled going to, say a true anomaly of $\pi/2$. Applying a thrust at $\pi/2$ also elongates the orbit, but only after traveling to that point along the original ellipse, which is the shortest distance from periapse. Plus, the shape of the orbit changes most dramatically at apoapse, causing the Hohmann Transfer ellipse and the flight path of Mars to intersect much sooner. The geometry of the problem is thus used to good advantage, in setting up the final mission profile.



The Hohmann Transfer Orbit from Earth to Mars

A spacecraft on an elliptical transfer orbit travels fastest at periape and slowest at apoapse. The segment of the trajectory from periape to a true anomaly of $\pi/2$ is not only geometrically efficient, as noted previously, but also energy efficient. Applying available thrust at $\pi/2$ increases the velocity on the slower part of the trajectory, compelling the spacecraft to reach the target much faster than if the same thrust were applied at periape. Applying this thrust anywhere before or after $\pi/2$ on the Hohmann ellipse increases the velocity less because more of the energy is going into changing the orientation of the orbit in space - which is helpful, but not as effective as spending all of this energy in elongating the orbit, which is what happens if the thrust is applied at $\pi/2$.

These simple principles of orbital mechanics indicate the efficacy of applying a major thrust at a true anomaly of $\pi/2$, as the best way to use a little extra thrust to do better than a straight Hohmann Transfer. Moreover, at the far end of the trajectory, the elongated transfer orbit turns out to have a more advantageous approach to Mars and the geometry at that point makes it possible to reach a final parking orbit around Mars more easily.

The transfer orbit has a true anomaly of about $\pi/2$ at approximately the same position as the Earth-Mars conjunction. Henceforth this $\pi/2$ point will be called "conjunction" and the thrust will be applied there, even though the final mission plan will not be exactly at conjunction.

Program Architecture

This paper is about trying to optimize an interplanetary trajectory without resorting to any sort of third party nonlinear optimization algorithms. However, there is nothing to prevent the completed trajectory - optimized though it may already be - from being further refined by a nonlinear routine. This is not a contradiction because if the program is structured right, additional degrees of freedom can be added, and the whole thing solved by a nonlinear problem solver. The simpler the base problem to be solved, the more degrees of freedom that can be handled by the nonlinear algorithm - keeping in mind that the interplanetary problem, without such strategies as noted above, is already too complex for a nonlinear program and can be solved only by the most general of all processes, the genetic algorithm.

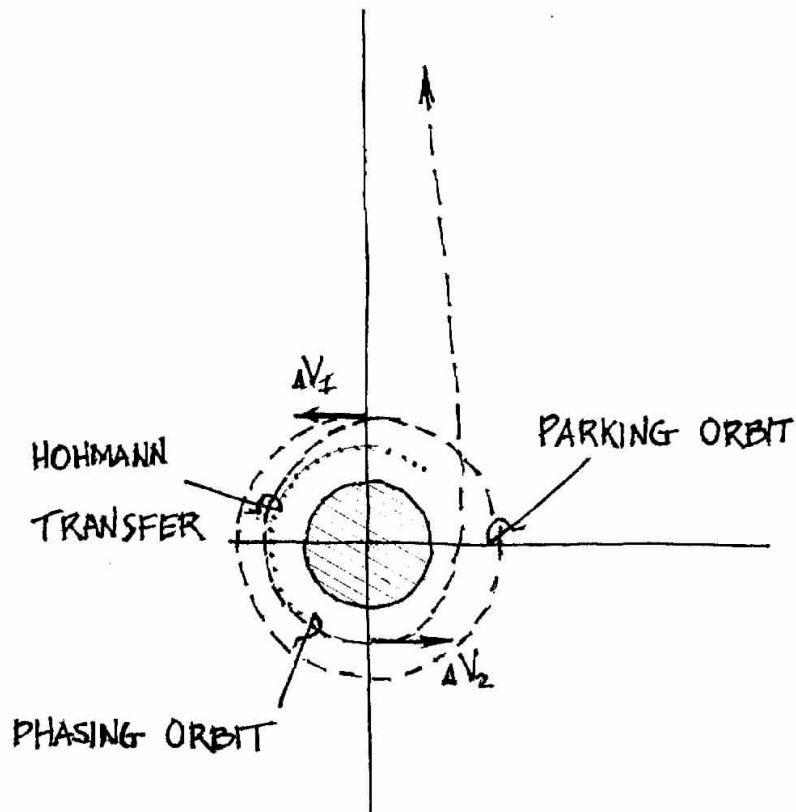
These are issues of programming architecture and mission design. Most Mars missions have a flight path correction right after leaving Earth's Sphere of Influence (SOI), to fine tune the long heliocentric ellipse to Mars. Similarly, there are typically thrusts just outside of Mars' SOI to set the spacecraft on the exact path for landing on Mars at a specific longitude and latitude. The mission design tool, developed here, has no such thrusts but the routine can easily be altered to add a thrust there, since the integration stops near each SOI for a change of coordinates. Other critical points are accessible for small thrust corrections like this, and are well suited for a nonlinear optimizer since they can be programmed with a confident range for equality and inequality constraints, such that all values in any given range will always generate a result.

It should be noted that the genetic algorithm approach is needed in so many interplanetary analyses because every single part of the whole problem is very nonlinear. Consequently, it is very important to establish a robust nominal trajectory, from which all variations in constraints result in solutions. The nominal trajectory, then, must be firmly grounded in fundamental orbital mechanics for it to have the kind of solution that can later be subtly modified at many intermediate points.

The choice of conjunction (e.g. a true anomaly of $\pi/2$) would seem to be a strong constraint on the problem, which being so inflexible would make the method seem much less an optimization. The rest of the routine, however, is programmed so that the thrust at the place called conjunction is nothing more than an acknowledgement that the spacecraft at

some point in transit from Earth to Mars is at a specific distance from the sun. The true anomaly at which this happens completely flexible, and the routine is free to apply the mid-course thrust (which can be set to equal zero) at the most efficacious point. Thus, specifying a priori that a major thrust is to be applied "at conjunction" is not a constraint at all, other than having the intention of applying a thrust in the general area of $\pi/2$ radians of true anomaly.

A last comment on programming strategy is to note that only central forces are simulated - e.g. no radiation pressure or atmospheric drag - in order to make it possible to do reverse integration. There is nothing wrong with adding these affects once the final optimized trajectory has been found, after which point the entire mission can be integrated straight from start to finish using the original code segments almost as is. The idea here is to leave such refinements to subsequent program and code refinements, and to focus here upon the basic trajectory and its numerical solution. Likewise, all thrusts are assumed to be point thrusts, while the code is always written to make it simple to modify for other types of thrust.



Hyperbolic Earth Escape from Parking Orbit

Two Point Boundary Value Problem

The nice thing about planning a priori a thrust near conjunction (and this value can be zeroed out by the user as an input) is that the mission is divided into two problems, conjunction-to Earth and conjunction-to Mars. Starting at a single point, the trajectory is integrated backward to Earth (the model having only central forces) and forward to Mars. The final "approach" to Earth is not necessarily at periapse for the Hohmann Transfer; nor the Mars landing at apoapse - as noted before, these are variables for the problem, which makes the starting point at conjunction a flexible quantity that is nothing more than a common point on the trajectory from Earth to Mars.

It is helpful to consider the whole algorithm at this point. A general time frame for a Mars mission is determined, and the exact ephemerides of the planets is used to find the coordinates for a true anomaly of $\pi/2$ on a two body Hohmann Transfer. The starting values of the problem are simply the epoch time of this position, with the spacecraft at the specified point on the Hohmann Transfer. The trajectory to Earth is first optimized from that time and position (in effect varying the true anomaly for an optimal "approach" to Earth, integrating back in time), often requiring a fractional thrust to be applied at conjunction. This thrust is applied to the original state vector, and from there the integration is completed to Mars. The Mars approach is a far more difficult problem to solve, even numerically, and requires a fractional adjustment to the time at conjunction. Once this part of the problem is solved, the state vector is adjusted at conjunction and the Earth trajectory is optimized again. At which point a complete, optimized trajectory is established.

The program accepts an input for thrust at conjunction, then finds the fastest flight path to Mars. Once an acceptable time of flight is determined, the starting time can be adjusted slightly to suit a specific mission plan - e.g. time of Earth departure. This is limited to a small window on the order of weeks, and the original optimal time solution is modified at the cost of extra thrust. These types of solutions, as slight variations from a robust nominal trajectory, would be the time when non-central forces could be added (integrating the entire trajectory forward in time, via a modified algorithm), and other such parameters as required by mission planning. The purpose of this report is to build a model to satisfy these requirements, in finding an optimal trajectory from Earth to Mars.

Coordinate Systems and Numerical Accuracy

The most numerically accurate way to calculate the trajectory is to integrate the motion with respect to the respective central bodies, and to add perturbing forces from third bodies. Always integrating versus central bodies makes the reverse integration possible as well. Another advantage of integrating versus central bodies is that barycenter calculations are not necessary, more so if a numerically stable equation is used for the third body perturbations. It is not difficult to show that this is the most accurate way, stopping the integration at SOI and changing coordinates. The third body perturbation is done using (Vallado 7-32)

$$\ddot{\vec{r}} = -\frac{Gm_E \vec{r}_{E\text{sat}}}{r_{E\text{sat}}^3} + Gm_3 \left(\frac{\vec{r}_{\text{sat}3}}{r_{\text{sat}3}^3} - \frac{\vec{r}_{E3}}{r_{E3}^3} \right)$$

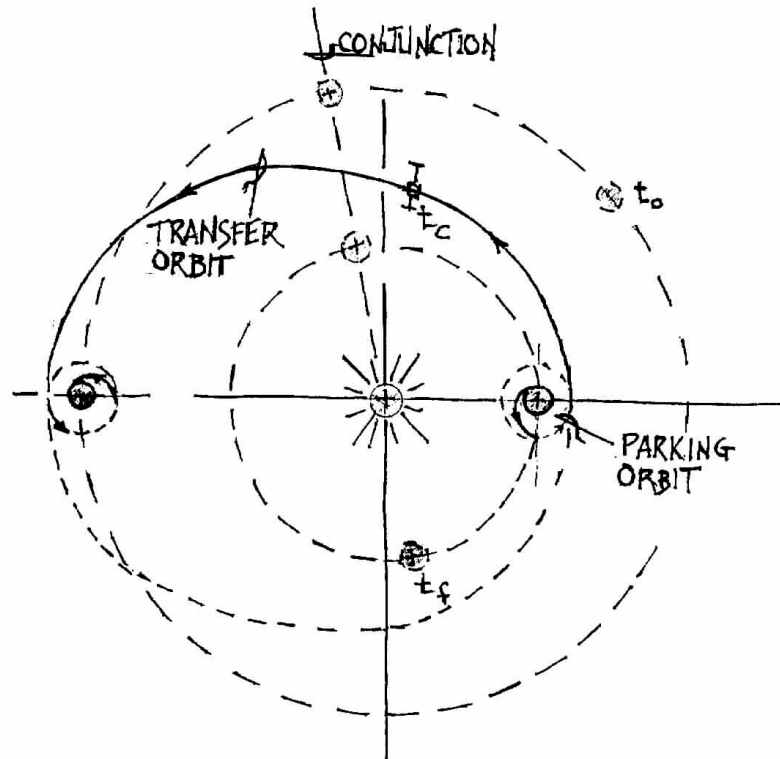
Although another, perhaps more accurate, equation is (Vallado 7-33)

$$\ddot{\vec{r}} = -\frac{Gm_E \vec{r}_{E\text{sat}}}{r_{E\text{sat}}^3} + \frac{Gm_3}{r_{E3}^3} \left(\vec{r}_{E\text{sat}} - 3\vec{r}_{E3} \left(\frac{\vec{r}_{E\text{sat}} \cdot \vec{r}_{E3}}{r_{E3}^2} \right) \right)$$

The code is set up so that additional perturbing bodies can be added, requiring a few more lines of code for each new body. The final approach within the SOI does not include any other perturbations other than from the sun, although these too could be included. There are several practical reasons for modeling the trajectory near Earth and Mars in coordinates centered at the respective planet. (There is no loss of numerical accuracy to do so.) Telemetry data near Earth - and now, near Mars as well - is available in local coordinates. Having the same units in the computer model avoids careless errors, makes last minute rush problem analyses and decisions easier, and allows seamless use of real time data by the spacecraft itself for autonomous decision making at critical points in the mission.

Conjunction to Parking Orbit

The trajectories to Earth and Mars are both first integrated from conjunction to the planet's Sphere of Influence (SOI). It is not difficult to target Earth's SOI, but Mars is a much smaller planet and the step size of the integrator must be fixed so that the routine does not go right past Mars (which is easy for the program to do because the step size is so big by that time). Once the SOI is achieved, there is a transfer of coordinates and the mission is continued in the local coordinates, to completion.



Hohmann Transfer between Parking Orbits

The key feature in the final part of the flight path after SOI is that the Runge Kutta 7/8 variable step integrator is used to select extremal trajectories. The integrator is designed to stop at every point at which significant changes have been detected in the problem - e.g. significant variations in the forces on the spacecraft, or changes in the parameters of the trajectory - so this saves the algorithm the trouble of having to make such a determination. This makes for a very fast and efficient routine, on the order of Artificial Intelligence because of how the integrator is able, in selecting the most sensitive points on the trajectory, to greatly expedite the convergence of the algorithm to a global minimum.

Every attempted approach trajectory must have a solution, or at least a numerical value, in an optimized routine like this. A one dimensional search routine is used to determine increments of thrust to apply at the starting point. When the central body is close, each stop of the routine calculates a two body Hohmann Transfer to the final parking orbit, and assigns this value to that point. When the trajectory is past the target, the minimum value is assigned to the flight path and the initial thrust at SOI is incremented accordingly, seeking a global minimum to the approach.

The flight path is modeled in this way so that the program can try and find a mission that takes advantage of gravity assist from the target planet. For Earth escape, a small gravity assist adds velocity to the spacecraft. For Mars capture, the "free return" profile of the desired trajectory helps to slow down the spacecraft ~ dramatically, if the algorithm is able to find the closed zero velocity curve for the sun-Mars system. This kind of approach profile is more probable, with a large reduction in thrust, with the kind of geometry in the flight path that happens with a major thrust at conjunction. This thrust not only gets the spacecraft to Mars much faster, but upon reaching Mars has a favorable geometry that saves just as much - if not more - thrust, in the final approach. Moreover, being a free return type of trajectory, the tragic pitfalls of a typical Mars landing are mitigated.

7 Gravity Assisted Bi-Elliptic Transfer Orbit

ABSTRACT

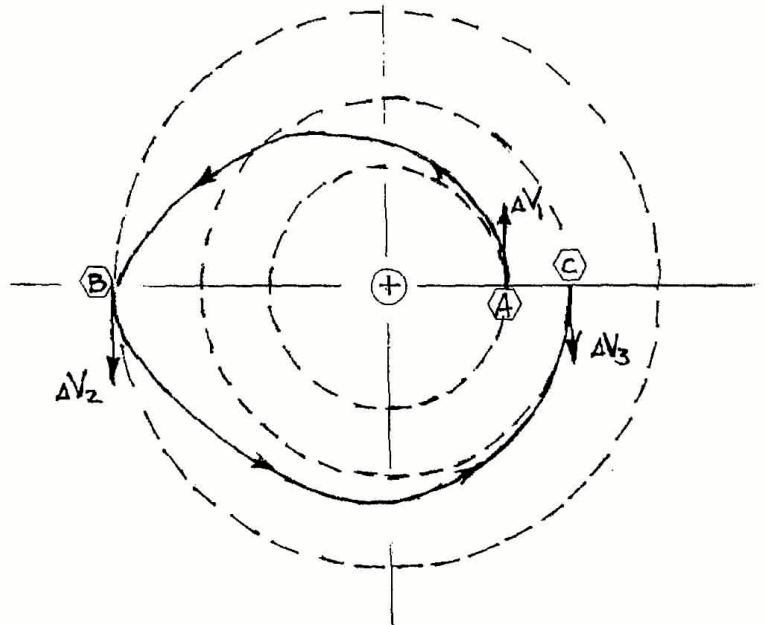
Bi-Elliptic trajectories are known to be more efficient than Hohmann Transfer orbits, although energy efficiency is typically achieved at the cost of a much longer time in transit. (see illustration below). This report proposes a bi-elliptic transfer orbit of a different kind. The first ellipse is identical to the first half of the Hohmann Transfer and the second ellipse is similar to the second half of the Hohmann Transfer, but with a shorter transit time and more favorable geometry upon approach to the target. Both trajectories are assumed to be a heliocentric ellipse. The program is designed to also take advantage of gravity assist "fly by" phenomena near each planet to make the overall trajectory even more efficient. Overall, the GRABET Transfer Orbit is faster and more efficient than the Hohmann Transfer. In the case of an Earth to Mars mission, the GRABET Transfer either arrives at Mars using 25% less fuel, 40 days faster, or something in between.

The GRABET Transfer orbit is intended to take advantage of nature's proven ability to find the fastest path between two points. The Hohmann Transfer is shown to be most efficient in the Two Body Problem (2BP) system, and this fact is used to initiate the GRABET Transfer by designating the Hohmann Transfer as the initial, nominal trajectory upon Earth escape.

The program then seeks the optimal point from which to begin the second elliptical orbit, using a thrust along some existing velocity vector. The objective is not a Lambert type transfer in which an exorbitant amount of thrust is used to make a rapid transfer to another orbit, but rather something intermediate between a Lambert and a Hohmann, minimizing energy use while making the transfer as fast as possible.

The advantage of using transfer ellipses that are slight variations off the minimum energy Hohmann Transfer is that the trajectory near the primary bodies, Earth and Mars, approximates the minimum energy paths nature predicates. Integrating the conjunction to Earth part of the trajectory, the spacecraft literally falls effortlessly - i.e. without the need for any flight path corrections - into the gravity well of the Earth. Integrating from conjunction to Mars, the spacecraft trajectory is adjusted until it finds the free return trajectory of the sun-Mars system, and the combined gravity of these two bodies moves the spacecraft into a

simple, robust approach path to Mars. Again, there is virtually no need for any trajectory correction maneuvers once this "free return" approach trajectory is achieved.



A Traditional Bi-Elliptic Transfer Orbit

The final approach to Mars is especially critical. The spacecraft is moving at half the speed of Mars, Mars being on a nearly circular orbit and the spacecraft being near the apoapse of a highly eccentric transfer ellipse, so the typical approach maneuver is to place the spacecraft in Mar's path and to wait for the planet to approach, for all practical purposes just like driving onto a railroad track and waiting for the freight train to come barreling down the tracks toward you. It's no wonder that 85% of all attempts to land on Mars have failed, crashing into the surface at the last moment ~ unable to make that delicate last second evasion into a safe parking orbit as Mars suddenly looms.

The GRABET Transfer orbit finesses the approach to Mars. The approach geometry is optimized to match the "free return" trajectory, and the program actually puts the spacecraft upon this flight path without the need for any flight path corrections near Mars at the very last moment, but just the single thrust at conjunction months before. The spacecraft then follows a heliocentric ellipse right to Mars, slips into the "free return" trajectory, and loops effortlessly around the rapidly approaching Mars along a trajectory that makes it virtually impossible to

crash into the planet. Instead, the spacecraft follows a graceful arc almost all the way around the planet at a constant altitude, that brings it along a "free return" path to an effortless loop around the sun. This loop can either be modified into a large ellipse for aerobraking into a high parking orbit, or adjusted for a low parking orbit and an expeditious maneuver to bring the spacecraft safely down to the planet surface.

In this instance, the notion of nature always finding the minimum energy path also includes the safest path, for the simple reason that crashing the spacecraft on Mars or missing Mars completely and ejecting out into deep space are the worse possible results, not only at the extreme of "minimum" but the total loss of all energy from the system. The difference between success and disaster is admittedly subtle, but the path itself is well defined in Three Body Theory as a closed zero velocity curve looping in a figure eight around Mars and the sun, with the crossing at a Lagrange Point.

Perhaps one day when Mars missions are routine, a small satellite will be placed in a halo orbit around the L2 Lagrange Point, giving approaching spacecraft a physical target; although it is likely that a series of such targeting orbits will be needed to ensure that the spacecraft is on the correct trajectory, matching the "free return" approach in both magnitude and direction of velocity. Something like the Babylon 5 "Jump Gate" architecture, and with a similar purpose - at least for approaching spacecraft. This would be especially important, since the L2 point may diverge from the Mars orbital plane (32).

8 Gravity Assisted Earth Escape Trajectory

ABSTRACT

The computer algorithm is set up to optimize each half of the flight path from conjunction to Earth (integrating backwards in time) independently. A fixed thrust is applied at conjunction, then the minimum thrust to reach parking orbit around Earth is determined. The "terminal" thrust is assumed to have three components: a large thrust to circularize at the targeting orbit, and two small thrusts to reach the final parking orbit via the Hohmann Transfer. Typically, the large thrust is combined with one of the two small thrusts, into one instantaneous thrust. All forces applied are along the velocity vector.

The numerical implementation of the "Earth capture" strategy necessitates the creation of a unique mapping in the vicinity of the target. Since the objective is to find a global minimum, it is essential that every thrust value applied at conjunction generates a total thrust to reach parking orbit. This total thrust need not be accurate to 12 significant digits except near the actual minimum. However, it must have a constant slope downward to the minimum from all directions so that a one dimensional line search can find the minimum easily. A cross section of the mapping provided later.

The energy/thrust "well" simulated by the algorithm returns a terminal thrust in such a way that it drives the final endpoint to the target quadrant. This is a point on the far side of the planet from which the s/c approaches, located at or near the final parking orbit itself.

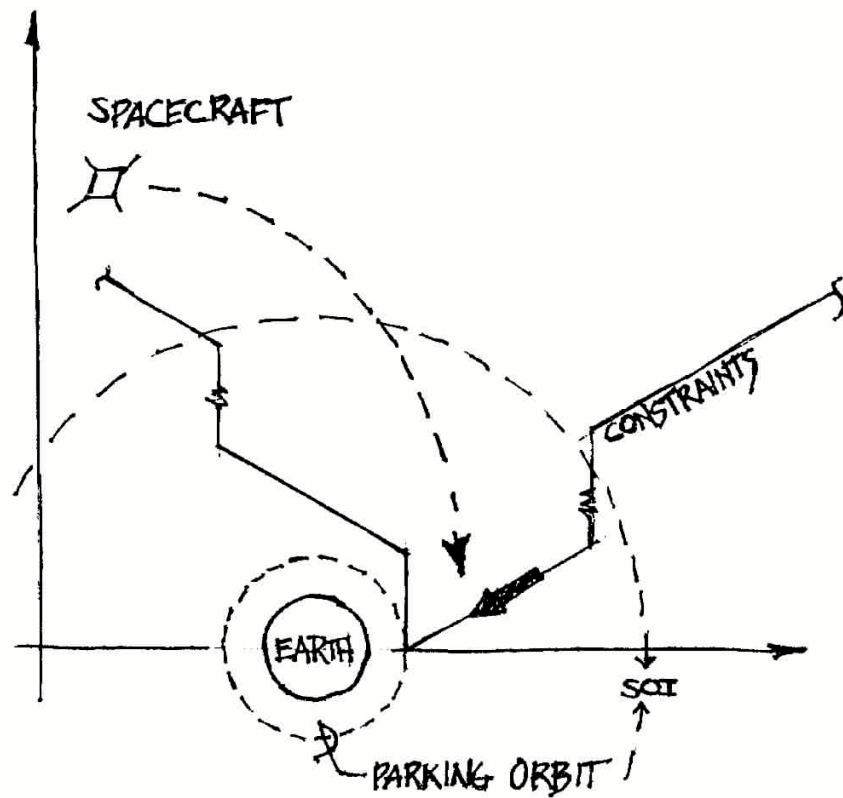
Conjunction to Earth

The technique for optimizing the conjunction to Earth flight path is simple. The initial state of the s/c is integrated back in time to Earth's SOI. Then the coordinates are changed from heliocentric to geocentric and the path is integrated until the range from Earth is increasing. The integration is done with a Runge-Kutta 7/8 variable step integrator. At each step the thrust needed to reach the 200 km final parking orbit is calculated (i.e. the orbit is circularized, then a small Hohmann Transfer is done to the final parking orbit).

It is assumed that the integrator stops for any significant changes in conditions on the spacecraft, thus no intermediate targeting orbits need to be considered. As the spacecraft

nears Earth, the lowest total thrust is saved in an array and when the range to Earth begins to increase, the loop is terminated and the minimum thrust scenario is retrieved from the array as the optimal trajectory.

After a single trajectory is optimized, the total thrust is compared to the total thrust of the previous run(s). If it is decreasing, the thrust at conjunction is decreased (and vice versa) and another trajectory is evaluated. The mapping is such that the global minimum is always in the direction of a negative gradient, so all the iterative loop must do is to conduct a one dimensional search until the minimum is reached within a specified tolerance.



Earth Gravity Well Target Mapping

9 The Free Return Mars Trajectory

ABSTRACT

The Mars capture trajectory is configured to seek a special type of incoming flight path called a "free return trajectory." That is, the Apollo missions to the moon were designed to approach the moon along a flight path that, if something went wrong, would bring the spacecraft right back toward Earth - no thrusts or maneuvers were needed for the spacecraft to return. It was a safety precaution, and it is also a very efficient trajectory.

The conjunction to Mars part of the trajectory is optimized by adjusting the velocity component of the initial conditions so that the trajectory ends just outside of Mars' orbit. Then, unlike how the conjunction to Earth flight path was configured, the initial conditions of the problem must be adjusted so that the spacecraft actually rendezvous with Mars. That is, the starting position of Mars is adjusted by a few days so that the spacecraft and Mars both arrive at the same time when their paths intersect near apoapse of the transfer ellipse.

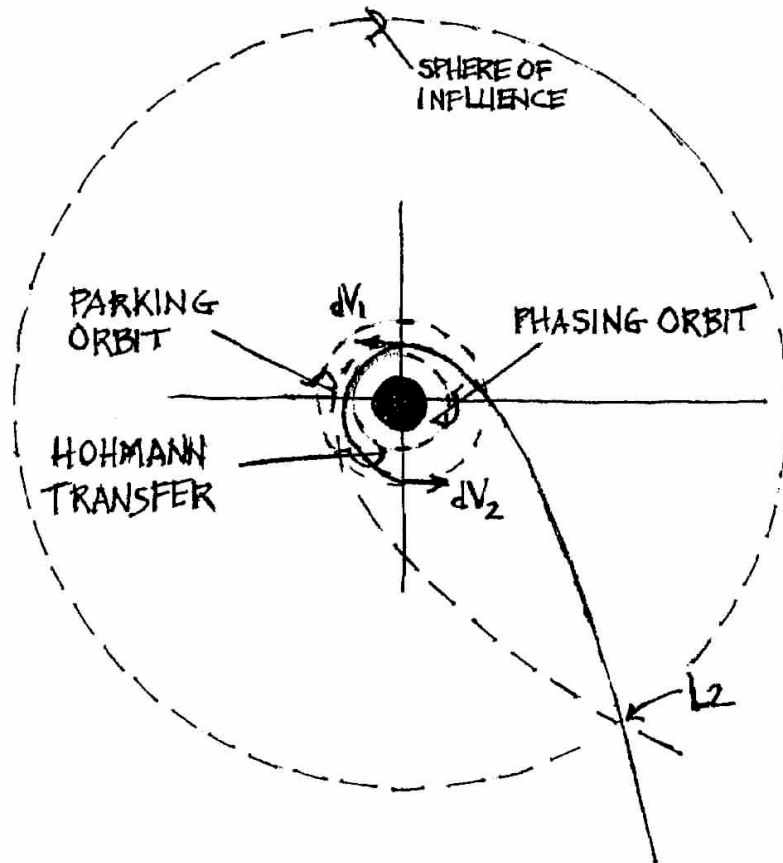
The first few iterations target Mars' Sphere of Influence (SOI). Subsequent iterations continue integrating the flight path until it either intersects Mars or goes beyond Mars.

The program output shows both stages of the process - the coarse, heliocentric targeting of the SOI, then the fine geocentric targeting of Mars itself. The integration is stopped as soon as the range to Mars starts increasing. This is the closest point of approach to Mars, and a tangential thrust is applied to put the spacecraft in a circular orbit. The spacecraft must still reach the 100 Km parking orbit, so additional thrust is added for a simple Hohmann Transfer to the lower orbit.

NOTE: In this "pure" Hohmann scenario, the optimal trajectory begins at the 100 Km parking orbit itself even though the optimization technique calculates an additional small Hohmann for prior iterations that do not terminate at the parking orbit; the program still needs a total thrust for each pass.

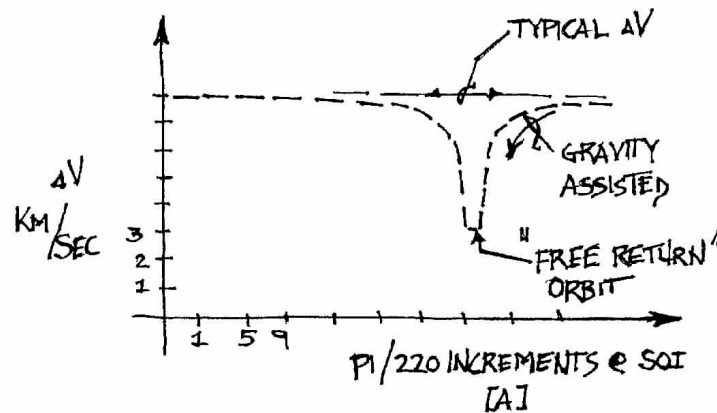
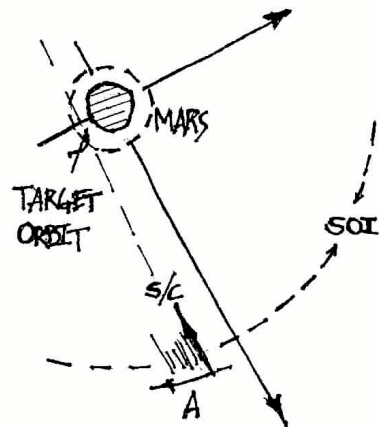
The user defined options allow more complex conditions. One scenario is for the spacecraft to go beyond Mars, to a higher altitude orbit, and to begin the final trajectory from there, slightly behind Mars. In this scenario, the spacecraft has a small gravity assisted flyby. Another option allows the periapse of this hyperbola to drop as low as 5 Km above Mars. In

either case there will be two thrusts calculated for the maneuver: The large thrust to enter orbit around Mars, and a second small thrust to go to the final 100 Km parking orbit.



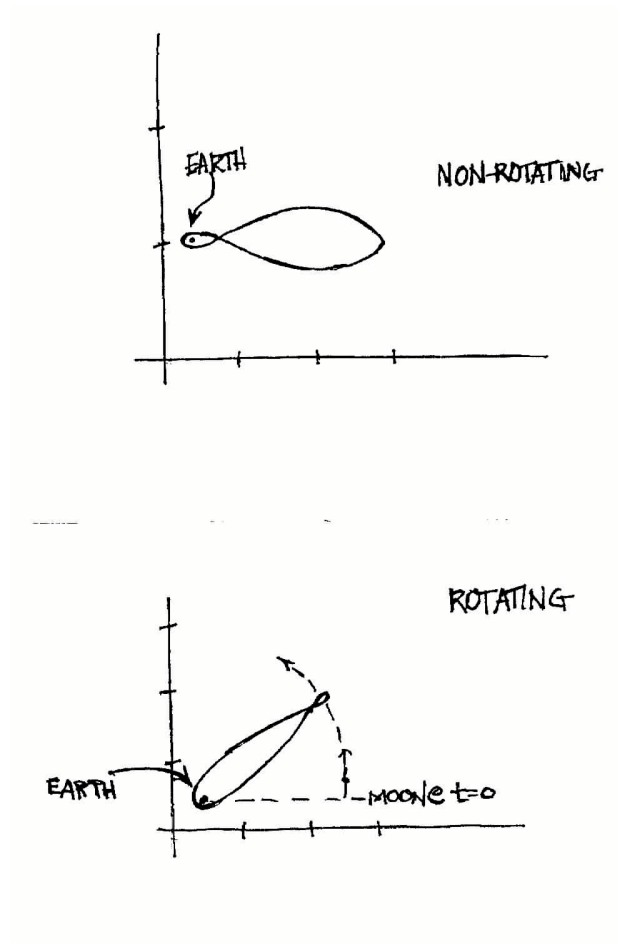
Mars Approach Geometry

The optimization of the user-defined scenarios is more complex than just stopping the integration when the range starts opening. These options are intended to explore the possibility of letting the spacecraft go closer to Mars to take best advantage of Mars' gravity in slowing down the orbit, and adjusting the geometry of the flight path so it's easier to enter a circular orbit around Mars. So, once the range begins opening, the program calculates the total thrust to reach the 100 Km parking orbit at each step of the integrator. The lowest final thrust among these values is saved as the global minimum is the optimal thrust for that particular pass. Additional passes are done until the closest point of approach to Mars on the trajectory is 100 Km (or less, as specified by the user). The lowest total thrust among all these passes is the global minimum for the Mars half of the flight path.



Targeting the Free Return Approach to Mars

The first objective is to adjust the phase angle of Mars at conjunction so the spacecraft path intersects Mars' SOI. This is a highly nonlinear problem and it cannot be solved by direct method used for the trajectory to Earth. Instead, once the state vector of the spacecraft is known at SOI, it is assumed that a small thrust can be done in real time to target any point on the SOI in the immediate neighborhood. Modifying the tolerances in the code can make the solution exact, at the cost of perhaps doubling the computational time. This was not done in the distribution copy of the program so the program can run on a PC in less than 60 seconds to get the approximate solution.

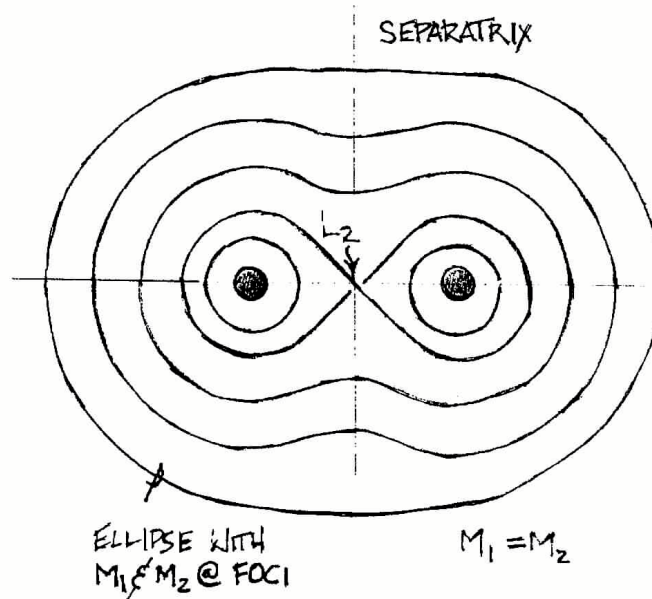


A Free Return Loop in the Earth-Moon System

The maximum possible error is 4 hours in the total time of flight of 260 days ($\pm .001\%$ ~ this decreases by at least one order of magnitude with each iteration, better if a sophisticated one dimensional search routine is used). The total thrust is not affected because of how the algorithm is structured.

Analysis

A phase path separating locally bounded motion and locally unbounded motion is called a separatrix (see the figure below). A separatrix always passes through a point of unstable equilibrium (e.g. the L2 Lagrange Point). Motion near a separatrix is very sensitive to initial conditions because points on either side of the separatrix have very different trajectories.



The L2 Lagrange Point as a Separatrix

The plot given is a zero velocity curve, for the Three Body Problem with equal masses. The potential function Φ is such that $\Phi = 0$ for such an equipotential surface. That is, the (gravity) field does no work on a body moving along an equipotential surface (e.g. a body on the L2 free return loop orbit between the two bodies will remain there indefinitely, with no other forces acting).

It is interesting to note that because the masses are equal and the paths are symmetric, it is possible to create a complete 3D map by rotating the above graph along the line shown. That is, the above map is identical for any section through the 3D map which includes both masses.

10 Mars Pathfinder Source Code

ABSTRACT

The interplanetary trajectory from Earth to Mars is difficult to solve numerically. Typically a problem with four or more unknowns requires a genetic algorithm to solve, the problem being intractable to any other nonlinear optimization methods. This paper shows how an Earth to Mars trajectory with twenty five unknowns can be solved directly, without the use of any nonlinear optimization methods whatsoever. The basic method finds a trajectory which is either thirty days faster than a flight path using the same total thrust as a Hohmann Transfer (i.e. the minimum energy trajectory in two body motion) or a trajectory which has the same time of flight as the Hohmann but uses 10% less total thrust. This report shows how the optimization algorithm is constructed and how it arrives at a solution using a relatively compact body of code that solves the problem to 12 significant digits in seconds on a personal computer.

The simplest and most efficient interplanetary trajectory is the Hohmann Transfer, the usual standard to which solutions are compared. The Earth to Mars trajectory, the topic of this report, had no analytical solution. The problem must be modeled on the computer and numerically integrated. A typical NASA mission has four thrusts: the initial thrust at Earth plus four Trajectory Correction Maneuvers (TCMs). At present, a simple trajectory with just two thrusts can be solved only with the benefit of a nonlinear optimization routine, whereas anything more complex must be solved in stages by genetic algorithm methods.

The patched conic method forms the basis for all interplanetary trajectory studies, by which the flight path is set up in specific configurations according to the forces acting upon the spacecraft (s/c). When the s/c is close to Earth, the central body is Earth and the Sun is a third perturbing body. This changes at the Sphere of Influence (SOI), after which time the Sun is the central body and Earth and Mars are perturbing bodies. (The SOI is the point where the force from the central body and a third body are equal in magnitude.) Then at Mars' SOI, the central force is Mars and the Sun is a third perturbing body. The transitions between planet centered and heliocentric coordinates is more than a convenience reference, but also the most accurate way to represent the forces from all perturbing bodies.

The trajectory to Mars from Earth is optimized in this paper without the benefit of any nonlinear optimization method. The two point boundary value problem with free time and position at both end points is separated into two independent problems with a common, fixed end point, approximately at conjunction. The algorithm then considers variations from the Hohmann Transfer. It converges directly to the solution. The method is accurate and fast, and generates an optimal solution that is either 10% better than the Hohmann in terms of total thrust or total time of flight.

The algorithm is compact and converges rapidly, the entire optimization taking less than thirty seconds on a 333 MhZ personal computer. This is many orders of magnitude faster than other methods of solution working on problems of a comparable difficulty. A total of 25 variables are optimized in this method. Such an algorithm would be useful for on board flight adjustments by a semi autonomous s/c, being able to perform real time calculations that currently can only be performed by mainframe computers at flight control on Earth. Also, many additional degrees of freedom can be added to this basic algorithm, and the entire problem then solved by third party nonlinear optimization methods. That is, for a problems with more than twenty five variables (e.g. a 3D simulation), this algorithm generates a nominal solution much closer to the optimum than the usual starting point, the Hohmann Transfer. Finally, the solution method is not abstract but easily visualized conceptually, giving the model and the algorithm a realistic aspect that makes it easy to imagine the forces, the bodies, and the overall characteristics of the entire problem. By comparison, the genetic algorithm is wholly statistical and uses a completely random way to arrive at the solution.

Establishing the Benchmark

As the computer model was being constructed, starting from a simple Hohmann Transfer between Earth and Mars in circular orbits, a standard was used to test the results before going on to the next, more complicated elaboration of the model. The standard used was the Hohmann Transfer, by which the s/c makes the transit from Earth to Mars in exactly 180 degrees of heliocentric longitude. Two thrusts are required, one at either end of the trajectory:

$$dV1 = 2.944 \text{ km/sec}$$

$$dV2 = 2.649 \text{ km/sec}$$

$$\text{total} = 5.593 \text{ km/sec}$$

$$\text{time} = 260 \text{ days}$$

The model solves the 2001 Earth to Mars trajectory, starting in a 200 km parking orbit at Earth and ending in a 100 km parking orbit at Mars. All motion is assumed coplanar with Earth and Mars are in their true elliptical orbits. Table 1 shows a range of values for a given TCM at conjunction.

	<u>DeltaV, km/sectime, daystotal</u>	<u>deltaV, km/sec</u>
1.2	214.224	6.764
1.1	216.133	6.511
1.0	218.019	6.308
0.9	220.063	6.157
0.8	222.766	5.779
0.7	225.131	5.645
0.6	228.339	5.625
0.5	231.510	5.283
0.4	235.749	5.381
0.3	240.580	5.202
0.2	247.730	5.131
0.1	262.190	5.157

Table 1 Trajectory Parameters versus the Hohmann Transfer

NOTE: The Hohmann Transfer time is 260 days and 5.593 km/sec

The Mid Course Correction

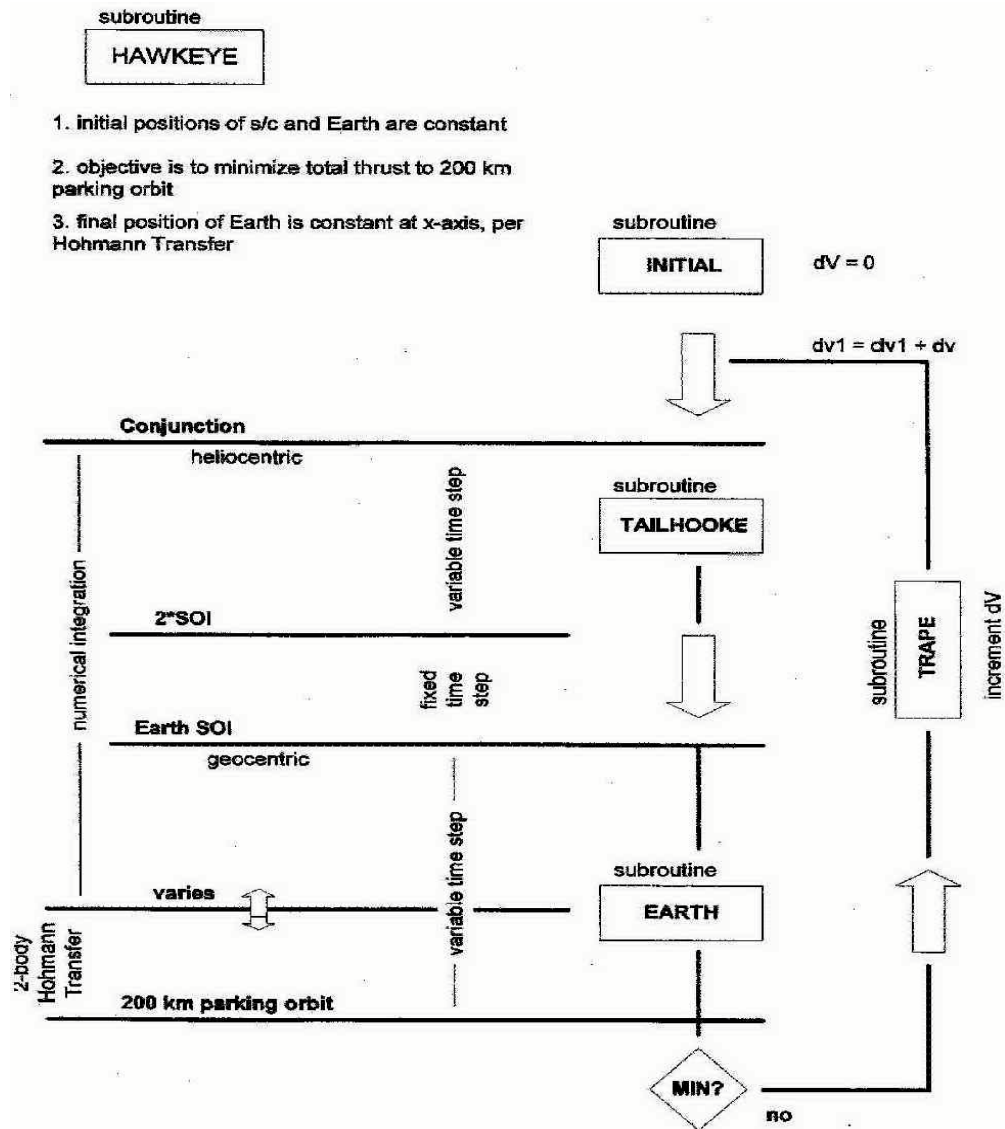
The unique aspect of the algorithm is that a major thrust is done at conjunction, or about half way to Mars from Earth. (Use of "conjunction" throughout this report is meant to signify an approximate location, at about 90 degrees from start in the trajectory, which coincides with the Earth-Mars conjunction in this particular situation.) There are several reasons why conjunction was chosen:

The s/c travels slowest in the second half of the trajectory, so the greatest benefit from a major thrust would be if this thrust were applied at conjunction

A thrust applied at a true anomaly of 90 degrees (e.g. conjunction for this particular problem is at a true anomaly of 94 degrees) acts to elongate the elliptical orbit. A thrust applied at any other point in the trajectory acts to rotate the orbit in space. An elongated orbit

is by far the best configuration as it causes the transfer orbit to intersect the path of Mars for a shorter time of flight.

The bi-elliptic transfer orbit is actually more efficient than a Hohmann, but is usually not used because it has a much longer time of flight. In this situation, separating the problem at conjunction and applying a large thrust there makes the solution into a bi-elliptic transfer orbit, where the geometry of both parts is favorable to the goal of finding a faster, more efficient trajectory.



Basic Formulation

The flight path from Earth to Mars has seven specific reference points, as follows:

1. A 200 km Earth parking orbit
2. The beginning of the interplanetary trajectory, near Earth but not necessarily at the initial parking orbit
3. Earth's SOI
4. The Earth-Mars conjunction
5. The end of the interplanetary trajectory, near Mars but not necessarily at the final parking orbit
6. A 100 km Mars parking orbit

The objective of the analysis is to optimize this fixed flight path between two points of constant energy, i.e. the designated parking orbits around Earth and Mars. The mission profile allows for a maximum of five thrusts. The optimal path begins with a small Hohmann Transfer from Earth parking orbit to a slightly higher orbit, then a main thrust to escape Earth gravity, followed by a mid-course correction at conjunction, and a small Hohmann Transfer at Mars into the final parking orbit around Mars.

It is assumed that a Hohmann Transfer can reach the intermediate "targeting" orbits most effectively, and that the thrusts for this planet centered trajectory can be approximated using two body equations of motion. The s/c enters the circular targeting orbit, from which it begins the integrated interplanetary trajectory with a large instantaneous thrust.

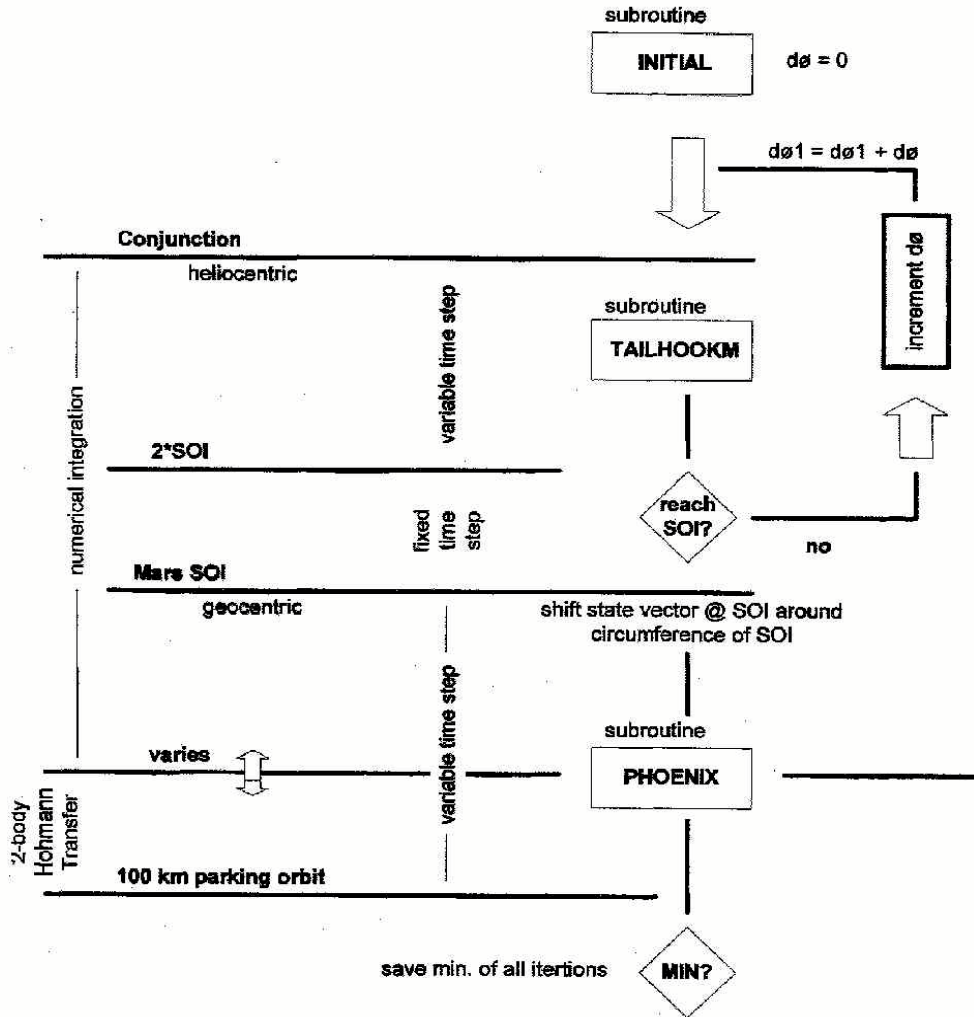
The flight path is integrated between the respective targeting orbit end points with three interruptions: one at each planet's SOI, and a third at conjunction. Within the SOI the integration uses planet centered coordinates and the sun as a perturbing third body. Otherwise, the coordinates are heliocentric with the motion of the s/c perturbed by Earth and Mars.

Throughout this analysis, all thrusts are aligned to the direction of motion. This is the most efficient thrust, transferring all energy to velocity and not wasting any energy on changes in the shape or orientation of the orbit itself. In so doing, all three major tangential thrusts increase the ellipticity of the orbit (for reasons noted earlier); again, to get the most out of the thrust in velocity.

subroutine

HAWKEYM

1. ΔV at conjunction is constant, $dV1$
2. vary phase angle at conjunction until s/c crosses Mars' SOI



Equations of Motion

The equations of motion are the standard Newtonian formulation for a gravitation force between two bodies. The integration is divided into two problems, both starting with the same state vector (coordinates and position) at or near conjunction. The motion of the s/c is integrated back in time to reach Earth (all forces are central forces, so integration with a

negative time step is OK), with a variable magnitude and direction to optimize both trajectories, independently.

The computer model is for elliptical, coplanar motion of the primaries using the J2000 ephemerides. The position of the primaries – i.e. Earth and Mars – is propagated using two body dynamical equations, with the inclination of Mars' orbit to the ecliptic set to zero. The integration is performed by a Runge-Kutta variable step (7/8) integrator set to a tolerance of $10E-12$, to give a consistent accuracy of 12 to 13 significant digits. The integration step is not altered externally with the exception of when the trajectory nears the SOI so the integration can be stopped as near to the SOI as possible to transfer the coordinates into a new reference system.

The flow charts will help to interpret the computer code.

```

C Last change: BC 15 Nov 2003 12:43 pm
c "Grabat" Orbits from Earth to Mars
c ~ Gravity Assisted Bi-Elliptic Transfer Orbits
c copyright 2003 william h. clark ii
c
implicit real*8 (a-h,o-z)
common/RKCOM/CH(13),AL(13),B(13,12)
CALL INRK78
CALL Earth (deltaV1)
thrust = 0.0d0
CALL Mars (thrust,deltaV2)
PRINT *, deltaV1 + deltaV2 ! = 0.24 km/sec < simple Hohmann
c Apply this thrust @ f=90 and arrive 30 days < simple Hohmann
STOP
END
c
SUBROUTINE Mars (thrust,dVmin)
implicit real*8 (a-h,o-z)
REAL*8 x(6),xe(6),xf(6),xm(6),xg(6),xtable(40,14)
common/conste2/xe,gmsun,gmearth,radE,re,soiE,ome,phE,rpE
common/constm2/xm,gmmars,radM,rm,soiM,omm,phM,rpM
pi = DACOS(-1.0d0)
CALL initial (t,x)
theta = -DATAN(x(5)/x(4))
c x(4) = x(4) - thrust*DCOS(theta)
c x(5) = x(5) + thrust*DSIN(theta)
tol = 1.0d-13
dt = 1000d0
do 1 n = 1,1500
CALL RK78T (t,x,dt,tol,6)
rangeM = DSQRT((xm(1)-x(1))**2+(xm(2)-x(2))**2)
IF (rangeM .LT. soiM*2) dt =1000.0d0

```

```

IF (rangeM .LT. soiM) GOTO 2
IF (dsqrt(x(1)**2+x(2)**2) .GT. rm) GOTO 2
1 continue
c transfer to geocentric coordinates
2 xg(1) = x(1) - xm(1)
xg(2) = x(2) - xm(2)
xg(3) = 0.0d0
xg(4) = x(4) + DSQRT(gmsun/rm)*DSIN(t*omm + phM)
xg(5) = x(5) - DSQRT(gmsun/rm)*DCOS(t*omm + phM)
xg(6) = 0.d0
soi= soiM
soiM = dsqrt(xg(1)**2 + xg(2)**2)
Vrel = DSQRT(xg(4)**2+xg(5)**2)
Rf = radM + 80.0d0 ! rpM
gamma = -pi/8.d0 + .0003d0
do 4 j = 1,40
gamma = gamma + pi/220.d0
xtable (j,1) = -soiM*dsin(gamma)
xtable (j,2) = -soiM*dcos(gamma)
xtable (j,3) = 0.d0
xtable (j,4) = 0.d0
xtable (j,5) = Vrel
xtable (j,6) = 0.d0
xtable (j,7) = 0.d0
xtable (j,8) = 0.d0
xtable (j,9) = 0.d0
xtable (j,10)= 0.d0
xtable (j,11)= 0.d0
xtable (j,12)= 0.d0
xtable (j,13)= 0.d0
xtable (j,14)= 0.d0
4 continue
gmsun = gmmars
gmearth = 0.d0
gmmars = 0.d0
tmax = 10.d0*soiM/Vrel
c
c do 78 m = 1,40
m = 27 ! tried all 40 cases; best results with this one
do 8 k = 1,6
8 xg(k) = xtable (m,k)
t = 0.d0
dt = +1000.d0
dVmin = 100.d0
Vrel = xtable(m,5)
do 75 n=1,1500
CALL RK78T(t,xg,dt,tol,6)
rangeM = Dsqrt(xg(1)**2 + xg(2)**2 + xg(3)**2)
IF (rangeM .LT. radM) THEN
deltaV = 100.0d0
GOTO 78
ENDIF
CALL velocityM (xg,gmsun,rangeM,Rf,Vrel,deltaV,V1,V2,V3)
IF (deltaV .LT. dVmin) THEN

```

```

do 10 k = 1,6
10 xtable (m,k) = xg(k)
xtable (m,10) = deltaV
xtable (m,9) = gamma
xtable (m,8) = rangeM
dVmin = deltaV
xtable(m,11) = V1
xtable(m,12) = V2
xtable(m,13) = V3
xtable(m,14) = t/86400d0
ENDIF
IF (rangeM .GT. soiM*1.1d0) GOTO 78
IF (t .GT. tmax) GOTO 78
75 continue
78 continue
RETURN
END
c
SUBROUTINE Earth (deltaV)
implicit real*8 (a-h,o-z)
REAL*8 x(6), xf(10)
step = 0.0001d0
ttol = 1.0d-7
ax = -.005d0
adelta = 100.0d0
CALL initial (t,x)
CALL TargetE (t,x,ax,adelta,xf)
bdelta = adelta
delta = adelta
2 DO 3 k = 1,3
IF (step .LT. ttol) GOTO 5
bx = ax + step
CALL initial (t,x)
CALL TargetE (t,x,bx,bdelta,xf)
IF (bdelta .LT. delta) delta = bdelta
IF (bdelta .GT. adelta) GOTO 4
ax = bx
adelta = bdelta
3 continue
4 bx = bx - step
step = step/2.d0
GOTO 2
5 deltaV = delta + bx
RETURN
END
c
SUBROUTINE TargetE (t,x,deltaV,deltaVf,xf)
implicit real*8 (a-h,o-z)
REAL*8 x(6),xe(6),xf(6),xm(6),xg(6)
common/conste2/xe,gmsun,gmearth,radE,re,soiE,ome,phE,rpE
common/constm2/xm,gmmars,radM,rm,soiM,omm,phM,rpM
theta = DATAN(x(5)/x(4))
x(4) = x(4) + deltaV*DCOS(theta)
x(5) = x(5) + deltaV*DSIN(theta)

```

```

tol = 1.0d-13
dt = -1000d0
do 75 n = 1,1500
CALL RK78T (t,x,dt,tol,6)
rangeE = DSQRT((xe(1)-x(1))**2+(xe(2)-x(2))**2)
IF (rangeE .LT. soiE) GOTO 78
75 continue
c change to geocentric coordinates (xe + xf = x)
78 xg(1) = x(1) - xe(1)
xg(2) = x(2) - xe(2)
xg(3) = 0.0d0
xg(4) = x(4) + DSQRT(gmsun/re)*DSIN(t*ome)
xg(5) = x(5) - DSQRT(gmsun/re)*DCOS(t*ome)
xg(6) = 0.d0
gm = gmsun
gmm = gmmars
gmsun = gmearth
gmearth = gm
re = -re
gmmars = 0.0d0
Rf = radE + 200.0d0
dt = -100.0d0
deltaVf = 100.0d0
do 85 n = 1,85
CALL RK78T (t,xg,dt,tol,6)
rangeE = DSQRT(xg(1)**2+xg(2)**2)
IF (rangeE .LT. radE) GOTO 88
IF (rangeE .GT. soiE*1.2) GOTO 88
Vrel = DSQRT(xg(4)**2+xg(5)**2)
IF (xg(2) .LT. 0.0d0) THEN
CALL VelocityE (xg,gmsun,rangeE,Rf,Vrel,DeltaVx,V1,V2,V3)
IF (deltaVx .LT. deltaVf) deltaVf = deltaVx
END IF
85 continue
88 gmearth = gmsun
gmsun = gm
gmmars = gmm
re = -re
RETURN
END
c
SUBROUTINE velocityM (xg,gmsun,range,Rf,Vrel,deltaV,V1,V2,V3)
implicit real*8 (a-h,o-z)
REAL*8 xg(6)
Rp = range
phi1 = datan(xg(4)/xg(5))
phi2 = datan(xg(2)/xg(1))
gamma = ABS(phi1 - phi2)
Vcirc = dsqrt(gmsun/range)
V3 =dsqrt( Vrel**2+Vcirc**2-2.d0*Vrel*Vcirc*dcos(gamma) )
V2 = dsqrt(gmsun/Rp)*(1.d0-dsqrt(2.d0/(1.d0+Rp/Rf)))
V1 = dsqrt(gmsun/Rf)*(dsqrt((2.d0*Rp/Rf)/(1+Rp/Rf))-1.d0)
deltaV = ABS(V1) + ABS(V2) + ABS(V3)
RETURN

```

```

END
c
SUBROUTINE velocityE (xg,gmsun,range,Rf,Vrel,deltaV,V1,V2,V3)
implicit real*8 (a-h,o-z)
REAL*8 xg(6)
Rp = range
Vcirc = dsqrt(gmsun/range)
V3 = dabs(Vcirc - Vrel)
V2 = dsqrt(gmsun/Rp)*(1.d0-dsqrt(2.d0/(1.d0+Rp/Rf)))
V1 = dsqrt(gmsun/Rf)*(dsqrt((2.d0*Rp/Rf)/(1+Rp/Rf))-1.d0)
deltaV = ABS(V1) + ABS(V2) + ABS(V3)
RETURN
END
c
SUBROUTINE thirdbc(t,n,x3)
implicit real*8 (a-h,o-z)
common/conste2/xe,gmsun,gmearth,radE,re,soiE,ome,phE,rpE
common/constm2/xm,gmmars,radM,rm,soiM,omm,phM,rpM
real*8 x3(6),xe(6),xm(6)
pi = DACOS(-1.0d0)
IF (n .EQ. 3) THEN ! position of Earth
x3(1) = re*DCOS(t*ome)
x3(2) = re*DSIN(t*ome)
x3(3) = 0.0d0
xe(1) = x3(1)
xe(2) = x3(2)
xe(3) = x3(3)
ENDIF
IF (n .EQ. 4) THEN ! position of Mars
x3(1) = rm*DCOS(t*omm + phM)
x3(2) = rm*DSIN(t*omm + phM)
x3(3) = 0.0d0
xm(1) = x3(1)
xm(2) = x3(2)
xm(3) = x3(3)
ENDIF
RETURN
END
c
SUBROUTINE initial (t,x)
implicit real*8 (a-h,o-z)
real*8 x(6),xe(6),xm(6),M,i
common/conste2/xe,gmsun,gmearth,radE,re,soiE,ome,phE,rpE
common/constm2/xm,gmmars,radM,rm,soiM,omm,phM,rpM
c input the planetary constants
gmsun = 1.32712428d11
gmearth = 3.986004415d5
re = 149598023d0 ! = a
ome = dsqrt((gmsun+gmearth)/re)/re
soiE = 924647d0
radE = 6378.1363d0
rpE = 200.0d0
gmmars = .305d4
rm = 227939186d0 ! = a

```

```

omm = dsqrt((gmsun+gmmars)/rm)/rm
soiM = 577213d0
radM = 3397.2d0
rpM = 80.0d0
c determine the Hohmann Transfer orbit paramenters
pi = DACOS(-1.0d0)
a = (re + rm)/2.d0
e = (rm - re)/(re + rm)
p = a*(1.d0 - e**2)
period = pi*(dsqrt(a)**3)/dsqrt(gmsun)
omsc = dsqrt((gmsun)/a)/a
tconj = (pi - period*omm)/(ome-omm)
phM = pi - period*omm
c start when f=90 for the spacecraft
EA = 2.0d0*DATAN(DTAN(pi/4)*DSQRT((1.0d0-e)/(1.0d0+e)))
M = EA -e*DSIN(EA)
t = M/omsc
x(1) = 0.0d0
x(2) = p
x(3) = 0.0d0
x(4) = -1.0d0*DSQRT(gmsun/p)
x(5) = DSQRT(gmsun/p)*e
x(6) = 0.0d0
RETURN
END
c
SUBROUTINE DERIV(t,x,f)
implicit real*8 (a-h,o-z)
real*8 x(6),f(6),xe(6),xm(6)
common/conste2/xe,gmsun,gmearth,radE,re,soiE,ome,phE,rpE
common/constm2/xm,gmmars,radM,rm,soiM,omm,phM,rpM
f(1) = x(4)
f(2) = x(5)
f(3) = x(6)
r2 = x(1)**2 + x(2)**2 + x(3)**2
r = dsqrt(r2)
f(4) = -gmsun*(x(1)/r)/r2
f(5) = -gmsun*(x(2)/r)/r2
f(6) = -gmsun*(x(3)/r)/r2
CALL thirdbc(t,3,xe)
CALL pert3b(gmearth,xe,x,f)
CALL thirdbc(t,4,xm)
CALL pert3b(gmmars,xm,x,f)
RETURN
END

SUBROUTINE energye(x,t,E)
implicit real*8 (a-h,o-z)
real*8 x(6),xb(6),xe(6)
common/conste2/xe,gmsun,gmearth,radE,re,soiE,ome,phE,rpE
r = dsqrt(x(1)**2 + x(2)**2 + x(3)**2)
E2body = 0.5d0*(x(4)**2 + x(5)**2 + x(6)**2) - gmsun/r
c
CALL thirdbc(t,re,ome,phE,xb)

```

```

r23 = dsqrt((xb(1)-x(1))**2 + (xb(2)-x(2))**2 + (xb(3)-x(3))**2)
Um1 = gmearth/r23
Um2 = ome*(x(1)*x(5)-x(2)*x(4))
Um3 = (gmearth/re**3)*(X(1)*xb(1) + X(2)*xb(2))
E = E2body -Um1 - Um2 + Um3
RETURN
END
C
SUBROUTINE pert3b(GMP,x3,x,f)
implicit real*8 (a-h,o-z)
real*8 f(6),x(6),x3(6),gmp,r12,r23
r12 = dsqrt(x3(1)**2 + x3(2)**2 + x3(3)**2)
r23 = dsqrt((x3(1)-x(1))**2+(x3(2)-x(2))**2+(x3(3)-x(3))**2)
f(4) = f(4) - gmp*((x(1)-x3(1))/r23**3 + x3(1)/r12**3)
f(5) = f(5) - gmp*((x(2)-x3(2))/r23**3 + x3(2)/r12**3)
f(6) = f(6) - gmp*((x(3)-x3(3))/r23**3 + x3(3)/r12**3)
RETURN
END

SUBROUTINE RK78T(T,X,DT,TOL,N)
IMPLICIT REAL*8 (A-H,O-Z)
common/RKCOM/CH(13),AL(13),B(13,12)
REAL*8 XD(21),F(21,13),X(N),F1(21),F2(21),F3(21),F4(21),F5(21)
c CALL INRK78 TO INITIALIZE
c IF(DABS(DT).LT.1.D-20)RETURN
TM=T
DT1=DT
C
DO 40 I=1,N
40 XD(I)=X(I)
GO TO 1020
C
1010 T=TM
DT1=DT
DO 41 I=1,N
41 X(I)=XD(I)
C
1020 CONTINUE
CALL DERIV(T,X,F1)
C K=2
DO 602 I=1,N
TP=B(2,1)*F1(I)
602 X(I)=XD(I)+DT*TP
T=TM+AL(2)*DT
CALL DERIV(T,X,F2)
C K=3
DO 603 I=1,N
TP=B(3,1)*F1(I)+B(3,2)*F2(I)
603 X(I)=XD(I)+DT*TP
T=TM+AL(3)*DT
CALL DERIV(T,X,F3)
C K=4
DO 604 I=1,N
TP=B(4,1)*F1(I)+B(4,3)*F3(I)

```

```

604 X(I)=XD(I)+DT*TP
T=TM+AL(4)*DT
CALL DERIV(T,X,F4)
C K=5
DO 605 I=1,N
TP=B(5,1)*F1(I)+B(5,3)*F3(I)+B(5,4)*F4(I)
605 X(I)=XD(I)+DT*TP
T=TM+AL(5)*DT
CALL DERIV(T,X,F5)
C
DO 50 K=6,13
KK=K-1
DO 71 I=1,N
TP=B(K,1)*F1(I)+B(K,4)*F4(I)+B(K,5)*F5(I)
IF(KK.LT.6) GOTO 71
DO 70 J=6,KK
70 TP=TP+B(K,J)*F(I,J)
71 X(I)=XD(I)+DT*TP
T=TM+AL(K)*DT
50 CALL DERIV(T,X,F(1,K))
C
DO 101 I=1,N
TP=0.D0
C TP=CH(1)*F1(I)+CH(2)*F2(I)+CH(3)*F3(I)+CH(4)*F4(I)+CH(5)*F5(I)
DO 100 L=6,13
100 TP=TP+CH(L)*F(I,L)
101 X(I)=XD(I)+DT*TP
C
IF(TOL.EQ.0.D0) GOTO 900
ER=0.D0
DO 112 I=1,N
A=DABS(X(I))
IF(A.LT.1.D-6) A=1.D-6
TEI=DABS(F1(I)+F(I,11)-F(I,12)-F(I,13))*CH(12) / A
IF(TEI.GT.ER) ER=TEI
112 CONTINUE
ER=ER*DABS(DT)+1.D-16
DT=DT*(TOL/ER)**.125D0
IF(ER.GT.TOL) GOTO 1010
C
900 CONTINUE
T=TM+DT1
RETURN
END

```

```

SUBROUTINE INRK78
implicit REAL*8 (a-h,o-z)
common/RKCOM/CH(13),AL(13),B(13,12)
DO 1 I=1,13
CH(I)=0.D0
AL(I)=0.D0
DO 1 J=1,12
1 B(I,J)=0.D0
CH(6)=34.D0/105.D0

```

```
CH(7)=9.D0/35.D0
CH(8)=CH(7)
CH(9)=9.D0/280.D0
CH(10)=CH(9)
CH(12)=41.D0/840.D0
CH(13)=CH(12)
AL(2)=2.D0/27.D0
AL(3)=1.D0/9.D0
AL(4)=5.D0/30.D0
AL(5)=5.D0/12.D0
AL(6)=1.D0/2.D0
AL(7)=5.D0/6.D0
AL(8)=5.D0/30.D0
AL(9)=2.D0/3.D0
AL(10)=1.D0/3.D0
AL(11)=1.D0
AL(13)=1.D0
B(2,1)=2.D0/27.D0
B(3,1)=1.D0/36.D0
B(4,1)=5.D0/120.D0
B(5,1)=5.D0/12.D0
B(6,1)=1.D0/20.D0
B(7,1)=-25.D0/108.D0
B(8,1)=31.D0/300.D0
B(9,1)=2.D0
B(10,1)=-91.D0/108.D0
B(11,1)=2383.D0/4100.D0
B(12,1)=3.D0/205.D0
B(13,1)=-1777.D0/4100.D0
B(3,2)=1.D0/12.D0
B(4,3)=1.D0/8.D0
B(5,3)=-25.D0/16.D0
B(5,4)=25.D0/16.D0
B(6,4)=1.D0/4.D0
B(7,4)=125.D0/108.D0
B(9,4)=-53.D0/6.D0
B(10,4)=23.D0/108.D0
B(11,4)=-341.D0/164.D0
B(13,4)=-341.D0/164.D0
B(6,5)=1.D0/5.D0
B(7,5)=-65.D0/27.D0
B(8,5)=61.D0/225.D0
B(9,5)=704.D0/45.D0
B(10,5)=-976.D0/135.D0
B(11,5)=4496.D0/1025.D0
B(13,5)=4496.D0/1025.D0
B(7,6)=125.D0/54.D0
B(8,6)=-2.D0/9.D0
B(9,6)=-107.D0/9.D0
B(10,6)=311.D0/54.D0
B(11,6)=-301.D0/82.D0
B(12,6)=-6.D0/41.D0
B(13,6)=-289.D0/82.D0
B(8,7)=13.D0/900.D0
```

```
B(9,7)=67.D0/90.D0
B(10,7)=-19.D0/60.D0
B(11,7)=2133.D0/4100.D0
B(12,7)=-3.D0/205.D0
B(13,7)=2193.D0/4100.D0
B(9,8)=3.D0
B(10,8)=17.D0/6.D0
B(11,8)=45.D0/82.D0
B(12,8)=-3.D0/41.D0
B(13,8)=51.D0/82.D0
B(10,9)=-5.D0/60.D0
B(11,9)=45.D0/164.D0
B(12,9)=3.D0/41.D0
B(13,9)=33.D0/164.D0
B(11,10)=18.D0/41.D0
B(12,10)=6.D0/41.D0
B(13,10)=12.D0/41.D0
B(13,12)=1.D0
RETURN
END
```

III. A Theory of F and G Forces

ABSTRACT

The overall objective of this project has been to find a faster, safer, cheaper mission to Mars. The computer simulation of the Earth to Mars trajectory found a flight path that is so much better than expected it was necessary to develop a formal theoretical basis for the trajectory, to justify the existence of the superlative results. In all things orbital, the most fundamental theory is the Three Body Problem (3BP) of Celestial Mechanics, and so that famous mathematical anomaly was investigated, leading to a possible explanation to the fast Mars trajectory. This explanation was not all together convincing, especially to experts in orbital mechanics. So this section takes a different approach, building a whole new theory which, not unexpectedly, leads right back to the 3BP.

Reconsider the solar system cosmology in light of the new developments in trajectory optimization. The planets were formed because of not just local dynamical influences, but also because of a "system wave" that differentiated the invariant (angular momentum) plane and the original symmetric plane, the latter being still active and is presumably some kind of galactic norm. (ONE)

In the same way, large and small forces were put to good use in seeking a minimum energy trajectory from Earth to Mars. The key turned out to be a very subtle phenomena of the sun-Mars system (e.g. existing in any such two body system) called a "free return" trajectory. This phenomena is what links two separate fractal regions - the solar system as a whole and Mars the planet's realm of local influence.

Fractal theory investigates a well known phenomena observed in so many different natural systems as to be assumed ubiquitous. Fractal theory is the idea that patterns repeat at subsequent levels of all natural systems. For example, the sun-Jupiter system is a Three Body Problem (3BP) and behaves just like the Earth-moon system, absent external influences. There is also a 3BP of quantum physics, on the subatomic level because the prevalent forces for gravity and electrostatics are both inverse distance squared forces. In theory, an atomic bond is analogous to the sun-Mars free return trajectory, like comets link our solar system to other solar systems in the neighborhood (or, at least, to a common center of mass).

Be that as it may, the question is how to quantify these phenomena. Fractal realms are individually well known, but next to nothing is known about the laws governing activity between the domains. (32) Discussion so far suggests that 3BP Theory is what links these realms, the free return trajectory being actually the plot of the lines of zero velocity or constant energy in the circular restricted 3BP.

Presumably a similar scheme governs the alleged transition between fractal domains formed by the invariant and symmetric planes. This is itself a very subtle thing - despite the scale of the solar system, and our intricate knowledge of it - so the analogous mechanism on the scale of a small satellite orbiting Earth would be much more subtle, if discernable at all. A first guess might be Relativity, but that has been explained away in the first section, (5) and will be further undermined by the first paper in this section. The answer seems to lie in the realm of Celestial Mechanics, but the answer is far from obvious. This section is devoted to a tentative avenue of investigation.

11 Relativity versus Celestial Mechanics

Most people don't realize that Celestial Mechanics has explanations for all the phenomena physicists explain using Relativity Theory. In fact, Relativity Theory was first derived in the context of Celestial Mechanics, by the famous French mathematician Poincare in the late 1800's. Einstein arrived at the same results by a different approach several decades later. In the early 1900's when Einstein was doing his best work, there were literally thirty experts in Celestial Mechanics who were his equal intellectually; if not smarter. . . These brilliant men were deriving applications beyond Relativity Theory long before physicists ever heard of the word - and we've been ahead of the curve ever since!

We're the mysterious "dark force" that astronomers say is causing all the objects in the Universe to move apart at an accelerating rate. Sure, they're moving apart when studied in the context of Relativity Theory. In the much more structured, mathematically exact science of Celestial Mechanics, the Universe is stable - it's the speed of light, "c" that varies. If physicists have recently managed to slow light down to zero velocity, how can any rational person assume that it's velocity is fixed at the other end of the spectrum?

Be that as it may, there are many nice things about Celestial Mechanics. It is much less abstract ~ you will see that 90% of the material on the Two Body Problem is basic geometry and trigonometry, and the rest is first year calculus. Celestial Mechanics is also far more generalized and with LOTS of room for more elaborate theories. We're still working on the Three Body Problem (which has no solution in closed form); I have some material on the Ten Body Problem (i.e. the Sun + nine planets); and there are also some comments on the Many Body Problem, which reduces down to the equations for an incompressible fluid. (This was proven over 100 years ago and is generally accepted by all astronomers.)

Celestial Mechanics shows that space acts just like a fluid at velocities on the order of the speed of light, not like Relativity's brick wall. This is quite obvious in solid state physics, which studies electric current that goes at a high percentage of the speed of light. Relativistic effects have never been observed in electric circuits; hence the "Relativity Barrier" does not exist either physically or mathematically. As an example, satellites in Low Earth Orbit exhibit relativistic effects in their ninety minute orbits; yet, electric current going around the world in a second, has no relativistic component. What more proof do you need?

Electric current is the flow of electrons, each one going a little distance before it releases its charge to the next electron. They all move in a "conduction band" at near the speed of light. Does electric current going across a continent exhibit exponentially increased

resistance, as Relativity says it should? No. Does the wire get much, much heavier, as Relativity says it should? No. Does the wire heat up - dissipating most of the electron's mass as energy - as Relativity says it should? No. Are scientists well aware of this paradox - do they strive to silence anybody who dares to speak of it? Yes.

Finally, there is the great victory that proved Relativity was better than Celestial Mechanics, Relativity's prediction of the precession of Mercury's perihelion. Consider the following table,

Planet	Calculated	Observed
Mercury	43.03"	43.11"
Venus	8.63	8.4
Earth	3.84	5.0
Mars	1.35	0.0
Jupiter	0.06	0.0

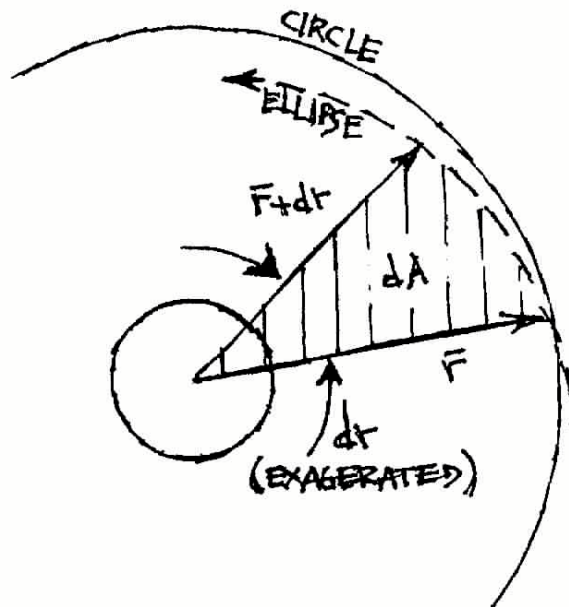
You can see that the "Calculated" correction by Relativity is not as accurate as everybody says, and in fact it gets worse as you go out from the sun. Relativity is based on the conviction that gravity propagates at the speed of light, a constant. However, this shows that the speed of light is not constant.

12 Kepler's 2nd Law

It is significant that, in developing the Universal Law of Gravitation (which leads directly to the Two Body Equation), Isaac Newton did not use the calculus. The Law of Gravity was derived exclusively from analytic geometry. He also used geometry to prove Kepler's 2nd Law, that a line from the central body to a satellite in orbit sweeps out equal areas in equal times. The analytic basis of this derivation begins with the common equation

$$(1) \quad dA = v r^2 d\theta = \text{"areal velocity"}$$

This function (using essentially $A = 2\pi r$) is not valid for ellipses, but for circles. Hence, Newton's derivation is not valid for elliptical motion. This is not hard to see from the geometry.



The Law of Equal Areas

Newton's proof looks at the two vectors in the limit as time goes to zero, at which point the length of the vectors is equal and the equation holds. However, if the two vectors are equal, *the orbital path is a circle*. Thus, this analytical proof is good only for circular orbits. Which leaves the physical evidence, that Kepler's second law is valid (e.g. via actual

observation of the planets). This discrepancy is not evident in any applications of this analytical proof, so there must be some aspect of the law itself that makes it work. The only possible explanation is that other, quite subtle, forces exist that make this law work in most situations. In extreme cases, discrepancies are attributed to Relativistic affects, but they are too nonspecific or generalized to apply to this situation. A more precise function is called for, and one that is much more intimately linked, specifically to orbital motion. Fortunately such a function already exists in the theory of orbits and it is also closely associated with elliptical orbits and the Two Body Problem (2BP).

An important equation in orbital theory is called the Two Body Equation.

$$(2) \quad \ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r}$$

which describes the acceleration (or force, from $\vec{F} = m\ddot{\vec{x}}$) of a small body orbiting a much larger body, e.g. a satellite orbiting Earth. This force is inversely proportional to the square of the distance r from the central body and it acts along the direction of the vector for r . The two other functions satisfy equation (2) and they are called the f and g functions.

$$(3a) \quad \ddot{\vec{f}} = -\frac{\mu}{r^3} \vec{f}$$

$$(3b) \quad \ddot{\vec{g}} = -\frac{\mu}{r^3} \vec{g}$$

These functions are used to propagate an orbit from an initial time $t(0)$ to some later time.

$$(4a) \quad \vec{r} = f\vec{r}_0 + g\dot{\vec{r}}_0$$

$$(4b) \quad \dot{\vec{r}} = \dot{f}\vec{r}_0 + \dot{g}\dot{\vec{r}}_0$$

A series representation in time can be made of the f and g functions, and it is used in initial orbit determination to calculate the orbital elements of a satellite just placed in orbit from three ranges determined to the satellite from ground radar stations. This method finds a new orbit plane for the observed data so that the orbital motion is according to the 2BP model. Thus the f - and g -equations are the transformation from one reference plane to another.

Now these f - and g -functions (equations 4a and 4b) are considered by the orbital mechanics community to be a mathematical curiosity, being able to propagate an orbit through time while also solving the Two Body Equation. It is helpful to pose this situation in the language of differential equations. Consider the Two Body Equation to be a special case of the general Ordinary Differential Equation

$$(5) \quad y'' + p(x)y' + g(x)y = 0$$

which can be linearized by making the substitutions

$$(6a) \quad y = \dot{r}$$

$$(6b) \quad \dot{y} = \ddot{r}$$

and this set of equations has two Linearly Independent (LI) solutions y_1 and y_2 such that the Wronskian has a nonzero determinant

$$(7) \quad \det \begin{bmatrix} y_1 & y_2 \\ \dot{y}_1 & \dot{y}_2 \end{bmatrix} \neq 0$$

The f and g functions are two such LI solutions, which in fact have a Wronskian identically equal to 1.

$$(8) \quad \det \begin{bmatrix} f & g \\ \dot{f} & \dot{g} \end{bmatrix} = f\dot{g} - \dot{f}g = 1 = \begin{bmatrix} f & \dot{f} \\ g & \dot{g} \end{bmatrix}$$

Notice that the following applies to equation 8 at apoapse and periapse.

$$(9) \quad \vec{r} \times \vec{v} = \begin{bmatrix} f\dot{r}_o & g\dot{r}_o \\ \dot{f}\dot{r}_o & \dot{g}\dot{r}_o \end{bmatrix} = \vec{r}_o \dot{r}_o = |\vec{h}| = const$$

It is interesting to notice that the two sets of eigenvector/eigenvalue pairs are quaternions. Furthermore, linear algebra says that a matrix of two LI vectors can be factored into an upper and lower triangular matrix - which implies there are two distinct mechanisms at work in the f/g factor translation of coordinates. This is consistent with the implications of previous sections, alleging an underlying system orchestrating the transformation between the symmetric and invariant planes.

Subsequent papers will show a physical basis for two new forces related to gravity. They do not act along a line as gravity does, but instead organize many bodies into a consistent reference frame. They are too subtle to be noticed in most situations, but are clearly evident when studying many bodies together. That is, in orbital mechanics, they are virtually invisible since analyses are already in a nearly exact inertial reference frame. It's only in a context of many massive bodies such as our Solar System, that these "f and g forces" can be seen.

It is easy to see that Newton's derivation of the Law of Equal Areas must have the sort of "relativistic" distortion at the origin evident with the system wave in order for the analysis to work for elliptical orbits. Hence, an indirect validation of the system wave concept.

Analysis

The f- and g-functions make the coordinate transformation as a matrix rotation - that is proven already. To do so the functions define a "symmetric plane" and does a pure rotation there. Here the invariant plane, in a Two Body Problem (2BP), is just the orbital plane. So these f- and g-functions define the relationship between the two planes in the 2BP as well as in the solar system, or 10BP. They are the equivalent of Relativity, but exist in all dimensions, not just in high gravity or high speed environments. It is significant that the 2BP is solved by the f- and g-equations in all dimensions ~ down to the Bohr Atom.

Presumably these equations define motion "in between" the orthogonal symmetric and invariant planes - which may mean when either plane is absent (analytically) or one is greatly predominant. They are kind of like a universal joint or ball bearing, except they are between planes and spaces and involve abstract math. They allow time to be invariant. The following study shows that the f-and g-functions behave exactly like the intercept of the symmetric plane at the origin.

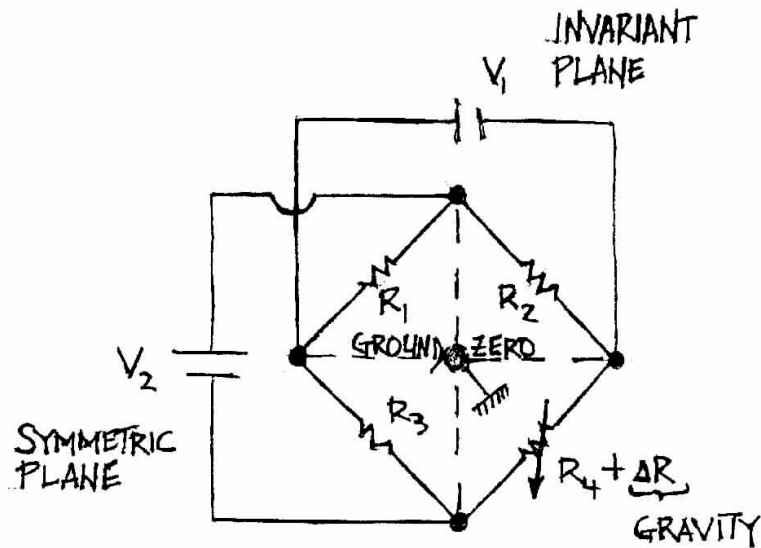
Begin with a heuristic look at Kepler's 2nd Law derivation.

$$\frac{dA}{dt} = \frac{1}{2}r^2 \frac{d\theta}{dt} = \frac{1}{2}r^2\dot{\theta} = \text{constant}$$

thus $\frac{dA}{dt} = \frac{1}{2}r^2 d\theta$ iff $r^2\dot{\theta} = \text{constant} = |\vec{h}|$

The conservation of areal velocity is not limited to an inverse squared law force (e.g. planetary motion) but is the general result for all central force motion, which includes 1/r forces.

The slight error in the mathematics that is picked up by Relativistic calculations also implies that $|\vec{h}|$ is not constant in direction. Thus the plane of the orbit has a slight wobble, just the kind of motion implied in the small eccentricity theory and exhibited at the origin of the symmetric plane. Before showing this mathematically, the electrical analog to the 2D harmonic oscillator can help to show how these new f- and g-forces interplay with the well known force of gravity. As shown in the theory of orbital mechanics in the study of motion near a planet, the f- and g-forces are all but nonexistent because, like a Whetstone Bridge, they balance the system so that it behaves just like a local inertial reference system; or at least independent enough so that fractal theory exists, but uniquely.



Electronic Analog of the Symmetric vs. Invariant Planes

It is interesting to note that, in fact, the force we know as gravity is actually quite small compared to the f- and g-forces, as suggested by the above illustration of a Wheatstone Bridge balancing circuit.

Now consider the mathematics of this situation, by reproducing the derivation for the f- and g-equations of celestial mechanics, but instead of the generalized case for any value of eccentricity or orbital shape; consider them for $e=0.01$ and $p=1$.

$$F = 1 - \frac{r}{p}(1 - \cos \theta) = 1 - r + \cos \theta = (1 - r) + \cos \theta$$

$$G_i = 1 = \frac{r_0}{p}(1 - \cos \theta) = 1 - r_0 + \cos \theta = (1 - r_0) + \cos \theta$$

$$FG_i = \cos^2 \theta + \cos \theta(1 - r + 1 - r_0) + (1 - r)(1 - r_0)$$

where the last term is $O(e^4)$ and can be neglected. The middle term cancels with a term developed later, both being something like $e^2 * [\sin \text{ terms}] * [\text{misc}]$ which are small and opposite in sign and $O(e^4)$

It was shown above that $FG_t = -GF_t$, so now calculate the second function and then compare the two sides.

$$G = \frac{rr_0}{\sqrt{\mu p}} \sin \theta = \frac{rr_0}{\sqrt{\mu}} \sin \theta$$

$$F_t = \frac{\sqrt{\mu}}{r_0 p} \left[\sigma_0 (1 - \cos \theta) - \sqrt{p} \sin \theta \right]$$

$$= \frac{\sqrt{\mu}}{r_0} \left[\frac{\bar{r}_0 \cdot \bar{v}_0}{r_0 p} (1 - \cos \theta) - \sin \theta \right]$$

$$= \frac{\sqrt{\mu}}{r_0} \left[\frac{r_0 v_0 \sin \phi}{r_0} (1 - \cos \theta) - \sin \theta \right]$$

Thus,

$$GF_t = r \sin \theta \left[v_0 \sin \phi (1 - \cos \theta) - \sin \theta \right]$$

$$= -r \sin^2 \theta + r v_0 \sin \phi (1 - \cos \theta) \text{ where } v_0 = r_0 \omega = r_0 \text{ and } \sin \phi \cong \phi \text{ (small)}$$

and substituting $r = 1 + e^2 \sin \theta + h.o.t.$ (higher order terms) you get

$$GF_t = -\sin^2 \theta + e^2 \phi \sin^2 \theta + h.o.t.$$

Given the \sin^2 and \cos^2 terms and their sum = 1 for all theta, then

$$\sin^2 + \cos^2 + [terms] = 1 \text{ for all theta, then the } [terms] \cong 0 \text{ for all theta.}$$

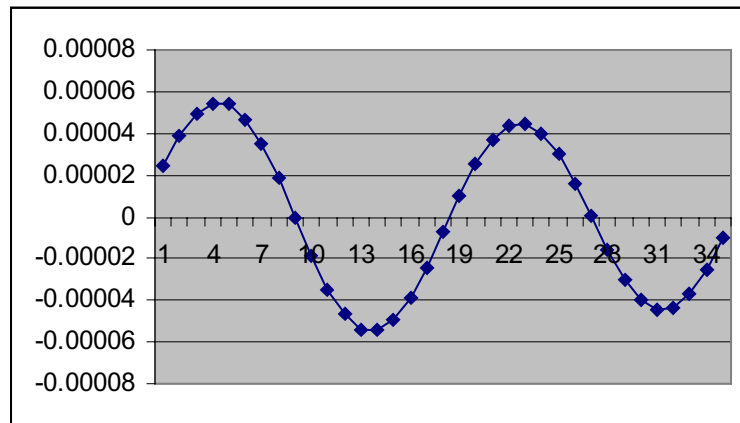
Now to show the action of one set of terms, and the other set of terms must do the exact opposite, in order for the determinant to equal zero under all conditions. Neglecting the cos term and the $O(e^4)$ last term, consider the other term

$$FG_t \Rightarrow \cos \theta (1 - r + 1 - r_0) \text{ recall } r = 1 + e^2 \sin \theta + h.o.t.$$

$$\Rightarrow \cos \theta (1 - 1 - e^2 \sin \theta + 1 - 1 - e^2 \sin \theta_0)$$

$$\Rightarrow -e^2 \cos \theta \sin \theta - e^2 \cos \theta \sin \theta_0$$

which works out to be an elongated sine wave (see graph below), just the motion that happens for the y-intercept of the symmetric plane at barycenter.



FG_t Remainder Term

This graph ignores the second term, assuming the coordinates are rotated to make the initial angle zero. The GF_t remainder term must be the exact opposite of this function so that the two terms sum to zero. One way to achieve this in 3D space is how the origin of the symmetric plane works so that near the origin there is a slight distortion but outside of a small neighborhood of the origin the symmetric plane is indistinguishable from any other flat plane in space.

13 The Complex Plane

The "real" plane analysis of gravity in the Two Body Problem showed there were three separate functions that satisfy the 2BP - r, f, and g. Some interesting mathematical support was offered, but in the end it was inconclusive. Now consider the same problem in a new coordinate system, the complex plane.

The Two Body Problem has an elegant solution in the complex plane.

$z = x + iy = R e^{-it}$ where e^{-it} is clockwise rotation

$$\begin{aligned}\dot{z} &= -i R e^{-it} + \dot{R} e^{-it} \\ &= iz + \dot{R} e^{-it}\end{aligned}$$

$$\begin{aligned}\ddot{z} &= -i\dot{z} - i\dot{R} e^{-it} + \ddot{R} e^{-it} \\ &= -iz - i(\dot{z} + iz) + \ddot{R} e^{-it}\end{aligned}$$

$$\ddot{z} = -2i\dot{z} + z + \ddot{R} e^{-it} \quad (1)$$

Notice how elegantly this representation breaks out the individual terms for the coriolis force, centrifugal force, and gravitational force. In Cartesian coordinates the only force was due to gravity. This suggests two actual forces exist in addition to the usual gravitational force. This analysis is for a circular orbit. Now consider the same analysis for an ellipse of small eccentricity. Consider a representation of an orbit of small eccentricity as follows:

$$R \cong R_p - a \cos t - b \cos 2t + \text{h.o.t.} \quad (\text{"higher order terms"}) \quad (2)$$

where $a = \frac{r_a - r_p}{2}$ and $b = a - p$ (the semi parameter).

Thus

$$\begin{aligned}\dot{R} &= a \sin t + 2b \sin 2t \\ \ddot{R} &= a \cos t + 4b \cos 2t\end{aligned} \quad (3)$$

These two equations are a Taylor Series expansion in sine/cosine about a unit circle in rotating coordinates. Substituting these into equation 1 you get a function in terms of R and its derivatives,

$$\begin{aligned}\ddot{z} &= R e^{-it} - 2i(-iR e^{-it} + \dot{R} e^{-it}) + \ddot{R} e^{-it} \\ &= -R e^{-it} + \ddot{R} e^{-it} - 2i\dot{R} e^{-it} \\ &= \ddot{x} + i\ddot{y}\end{aligned} \quad (4)$$

That is,

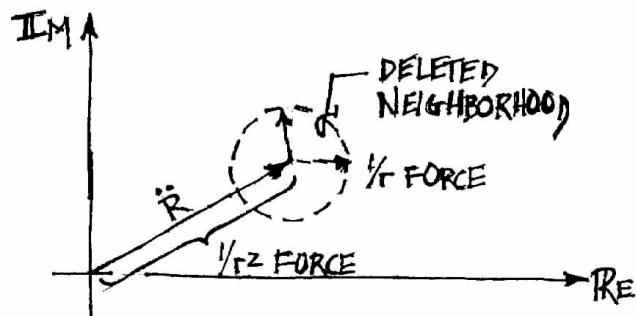
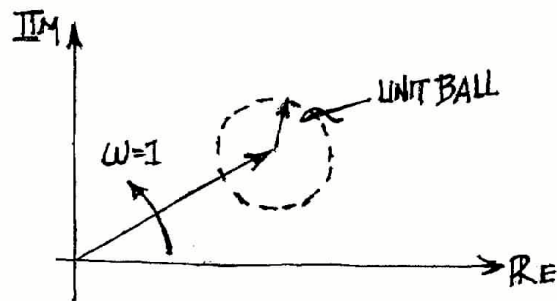
$$\ddot{x} = -R e^{-z} + \ddot{R} e^{-z}$$

$$\ddot{y} = -2\dot{R} e^{-z}$$

(5)

which are both simple functions of t only while both a and p are slow variables, that is $\dot{a} = 0$ and $\dot{p} \approx 0$. The total force is a function of three terms again - R , \dot{R} and \ddot{R} . Also observe that, like the f and g functions, a series representation is possible of these forces in the complex plane using trigonometric series for sine and cosine.

In equation 4, the first term is motion on the unit circle (for the normalized or regularized dynamical system) and the last two terms are a vector in the complex plane with both vectors rotating at the same frequency (i.e. an octurnion).



Distorted Space-Time in the Deleted Neighborhood

Posed in terms of the forces, it might fit better to the f- and g-force hypothesis and also with the series representation of gravity - being comprised of a fundamental frequency (the unit circle) plus a perturbation. In this context, the orbital motion is centered at the center of the ellipse (not at one focus), which is how the $1/r$ force works. This is just how the symmetric planet was proposed to act.

It is appropriate to elaborate further on the complex/symmetric plane analogy (in order of decreasing strength of correspondence)

both represent orbital motion as perturbations from the unit circle

derivatives of all order exist for the equations of motion

both are coordinate systems rotating at a constant rate, normalized to the unit circle

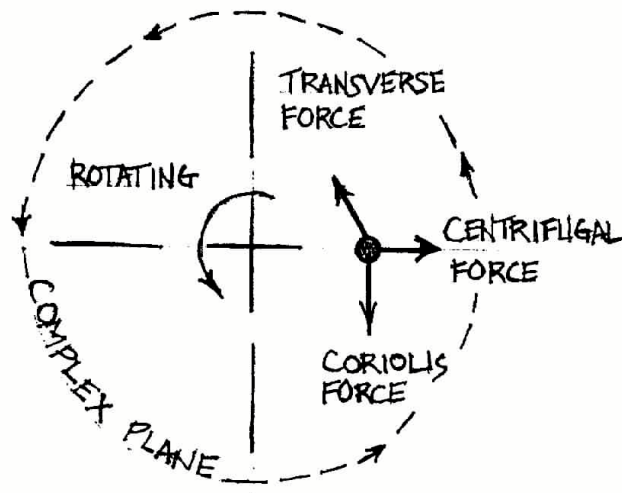
contour integrals have the same action as zero velocity curves

the complex integral and the Jacobi integral are equivalent

both have a reflection principle

This strongly suggests that the symmetric plane of the solar system is, in fact, the well known complex plane and that the difference to the invariant plane is due only to the mathematical representation of the origin - barycenter versus geometric center, new versus old. Barycenter is a point singularity, while the symmetric plane avoids this as a coordinate system origin by simulating a "deleted neighborhood" there, a small 3D sphere.

In other words, in order to solve the unrestricted general 3BP it is first necessary to transform the coordinates into a symmetric plane.

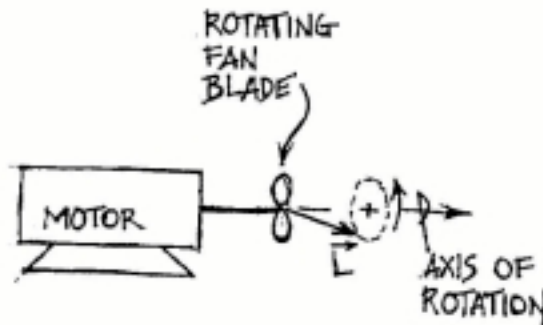


Forces on a particle in a Rotating Coordinate System

Analysis

The above illustration shows the forces acting on a bug crawling along a radial line on a rotating turntable.

- Coriolis Forces act only when a particle is moving in a rotating coordinate system and always act perpendicular to the direction of motion. On Earth, these forces are what cause the circulation of air around high/low pressure systems.
- Transverse Forces are perpendicular to the radius vector, but occur only if the rotating coordinate system is accelerating (which is not the case in the Three Body Problem scenario, which assumes a constant rate of rotation). Later it will be shown that this is the displacement force causing tesseral harmonics.
- Centrifugal Forces are due to rotation about any axis and act outward along the radius vector.

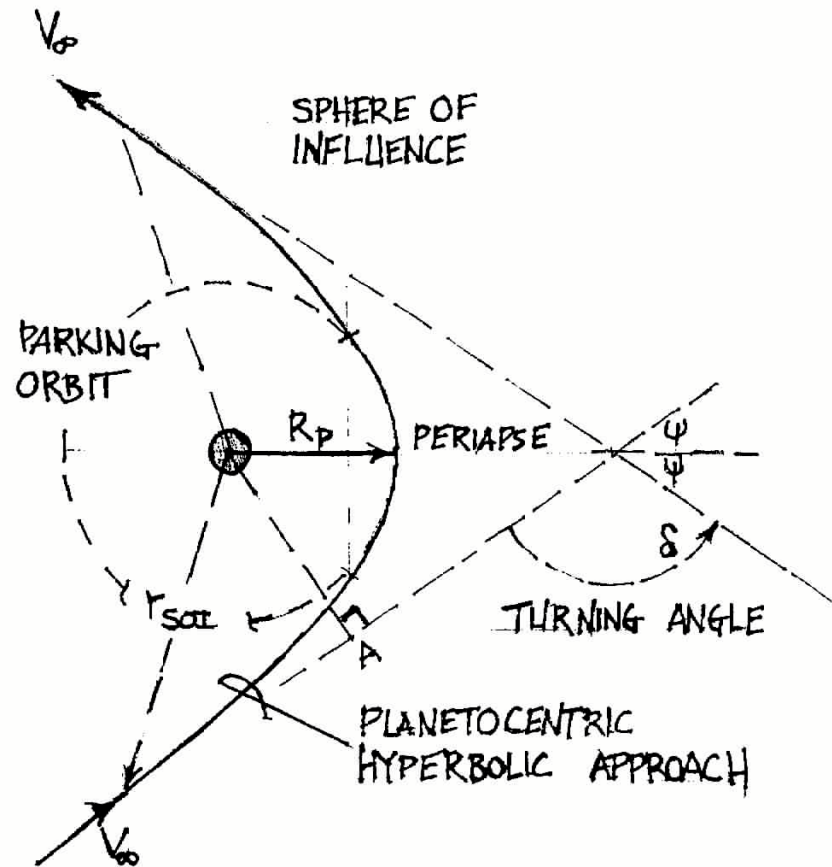


Planet's Halo Orbit vs. its Axis of Rotation

The above illustration is a dynamical model of the L6 Lagrange Point that defines the planet's motion versus the symmetric plane as a figure-8 loop (the fan blade) while a slight "imbalance" in the motor shaft (the difference between the invariant and symmetric planes) causes the planet's angular momentum vector to describe a small cone about the nominal axis of rotation. This dynamical imbalance happens because the axis of rotation of the fan is not exactly a principal axis of the system. This is how planet's rotation brings the whole system back into a dynamic equilibrium.

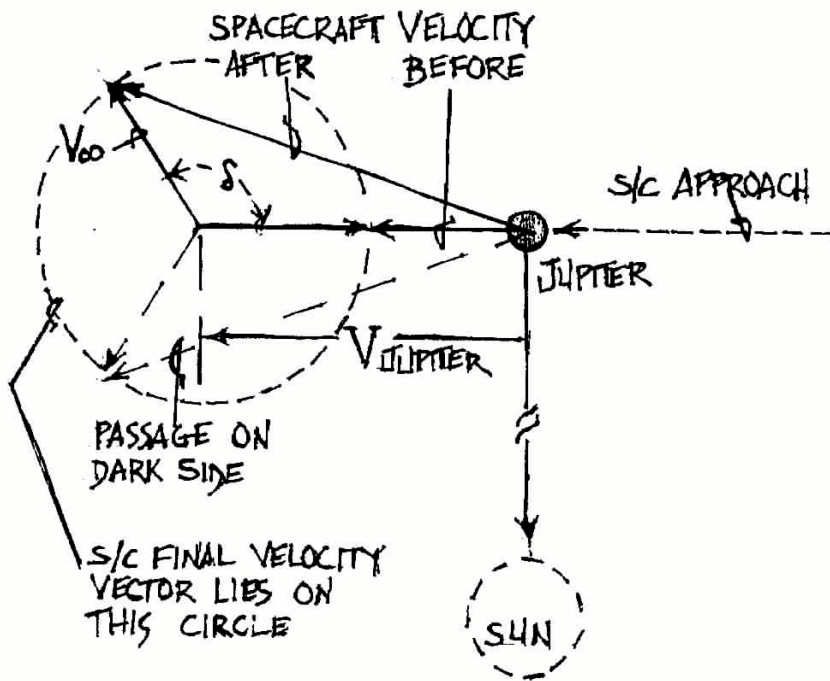
You will recall, per the uppermost illustration, how this deleted neighborhood is related to the f - and g -forces (12) where the action of a second $1/r$ force is superimposed upon the regular inverse squared gravity field. The resultant action, considering the center of

the unit ball to be the L6 Lagrange Point, is exactly the geometry of a halo orbit around its Lagrange Point in a rotating coordinate system. In non-rotating coordinates, the halo orbit is just a regular elliptical orbit. This geometry is further delineated in the geometric study of the hyperbolic fly by or gravity assisted fly by of a planet or large mass.



Planet Fly By Geometry

A generalized solution substitutes an ellipse for the circle (unit ball), and this ellipse is centered at the end of the unit vector as shown (i.e. a circle at some angle to the plane of the page). This is the classical $1/r$ force system characteristic of all halo orbits. Having no body at the focus, for halo orbits still, the $1/r$ force is the only possible way to quantify an elliptical orbit about (and whose center is) the Lagrange Point.



14 The Szebehely Equation

The following study suggests that the solar system behaves just like three bodies - i.e. the equation developed for the "system wave" solves the equation for a regularized 3BP. The three bodies presumably are the sun, the inner planets, and the outer planets. You may recall that all the solar systems discovered around other stars have a massive sun plus just one or two gas giants, bigger than Jupiter.

Consider the equation for the system wave, where in the case of fitting the position of the planets to some common waveform, the value of a in the helix equation was close to zero. From the analytic geometry, when $a=0$ the solution is the unit circle. Consequently, the planets in this new scheme follow orbits of small eccentricity and it takes their combined influence to collectively avoid the singularity of a unit circle. The mathematics of the situation follows.

$$\begin{aligned} y &= a \tan t + \sin t = \sin t \left(\frac{a}{\cos t} + 1 \right) \\ &= \tan t (a + \cos t) \cong \sin t, a \searrow 0 \\ x &= a + \cos t \cong \cos t, a \searrow 0 \end{aligned}$$

Which means motion is restricted about the unit circle, and although x and y are not identically equal to the sine and cosine terms respectively, their derivatives can be accurately represented as

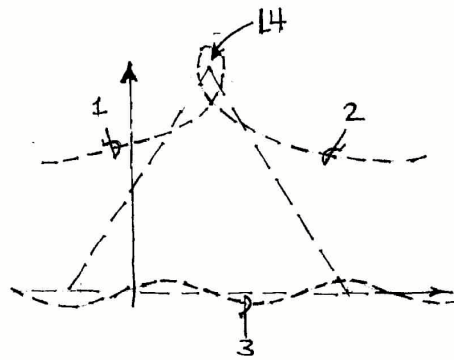
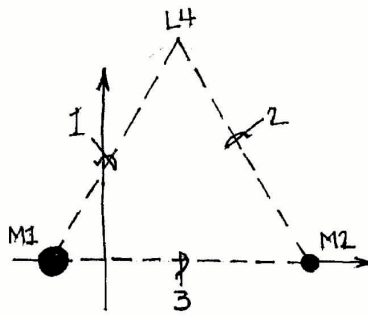
$$\begin{aligned} \dot{x} &= -\sin t \\ \dot{y} &= \cos t \end{aligned}$$

Now consider the Jacobi constant for this rotating system,

$$\Lambda^2 = \dot{x}^2 + \dot{y}^2 = \sin^2 t + \cos^2 t = 1$$

from which $\Lambda_x = \Lambda_y = 0$ because Λ is constant for the circular coplanar 3BP. Notice that $\Lambda = 1$ is motion on the unit circle but $\Lambda < 0$ for motion to exist - i.e. rotation - so motion on the unit circle is not possible; it's a singularity. If you evaluate the problem in the complex plane, then

$$y = j \sin t$$



Stability Points in the Ten Body Problem

Now consider the Szebehely Equation for the restricted 3BP

$$\ddot{y} = \frac{1+y^2}{\Lambda} \left[\Lambda_y - \dot{y}\Lambda_x \pm 2\sqrt{1+y^2} \right]$$

Substituting for Λ ,

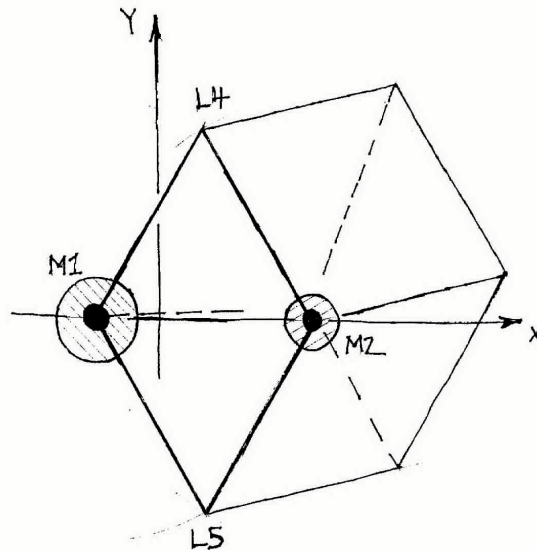
$$-j \sin t = \frac{1 - \cos^2 t}{j} [\pm 2 \sin t]$$

$$\sin t = \sin^2 t [\pm 2 \sin t]$$

$$\pm \frac{1}{2} = \sin^2 t \Rightarrow t = \frac{\pi}{4}$$

To put this into context, consider the earlier statement that the fact that the system wave "solves" the Szebehely Equation means the planets of the solar system are in some context organized as a 3BP. The system wave places the Earth near an equilibrium stable point and the moon in a stable orbit around this point. (3) The graphical similarity implies, as expected, that each planet orbits a stability point on the system wave, like a halo orbit in the 3BP. (It is

likely these stability points also exist in the 3BP.) Notice that the 10BP rotates into the plane of the page, while the 3BP rotates in the plane of the page. Thus lines in the 10BP will be points in the 3BP, and the equilibrium stability points in the 10BP will be orbits in the 3BP - theoretically.



Projecting the Equilibrium Points into Three Dimensions

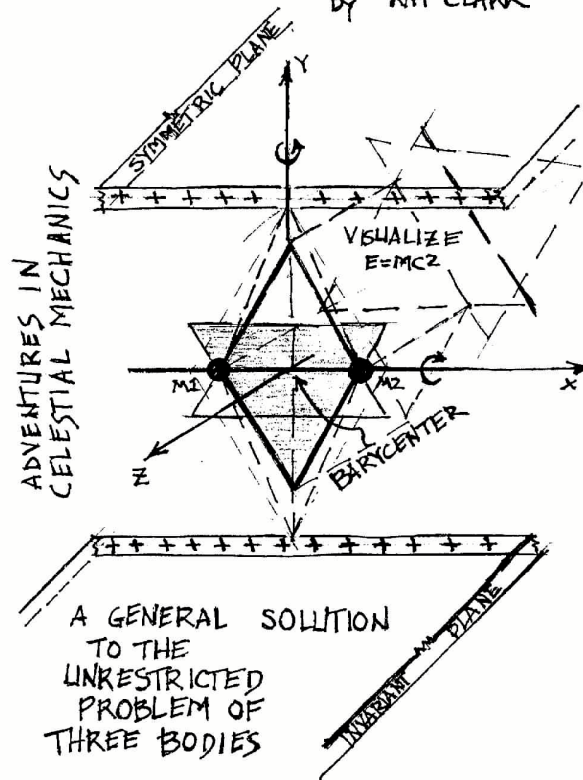
It was suggested before that the Szebehely Equation, or the system wave, is the transition between fractal levels, which apparently alternate coordinate systems which are perpendicular to each other. The only configuration of stability points - if they are to be considered an orthogonal/independent and mutually exclusive set, level to level - is for them to be the corner of a box in a 3D transition space. Thus the Lagrange Points in the transition coordinate system are at an angle of $\pi/4$ as derived above.

Analysis

How to analyze the Szebehely Equation as a dynamic system when the analysis versus the system wave has been of a virtually static system? Even the projection of the whole body of work into three dimensions (refer to the next figure) is static, if complicated. It is important to establish motion because this shows how the system changes over time, evolving from a cosmic cloud into a stable system of planets, as it is now. Not even Relativity has a mechanism for change, having instead a fixed cosmic constant.

GRAVITY GAMES

by KH CLARK

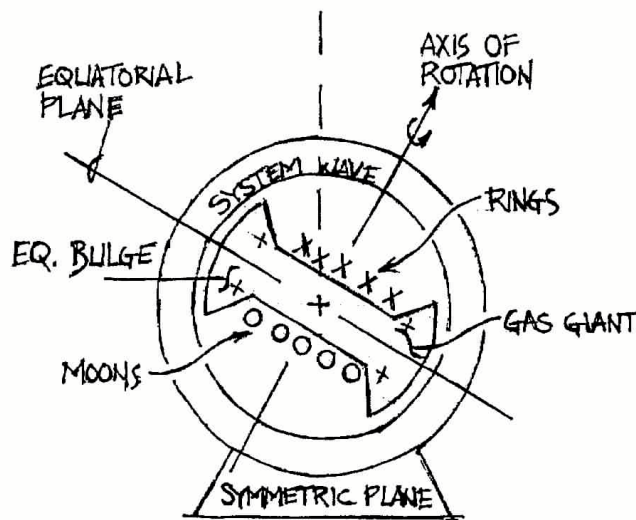


The Three Body Problem in the Context of Relativity Theory (find the box of the previous figure)

It is helpful to return to the motor/generator analysis, which served so well (3) to conceptualize the complex interaction of gravitational and electromagnetic fields. Consider now a six pole stator, but now with windings on the rotor as well. That is, the analysis so far has been static - focusing upon the stator, which is unchanging - so now that a dynamical element to the system is sought, it is logical to look at the role of the rotor in the electromechanical analogy. In this case, the rotor moves but so slowly in terms of Earth orbits or years, as to be practically imperceptible. Rather this rotor turns infinitely slow so that only over the course of the evolution of the solar system does the system behave like a motor/generator set.

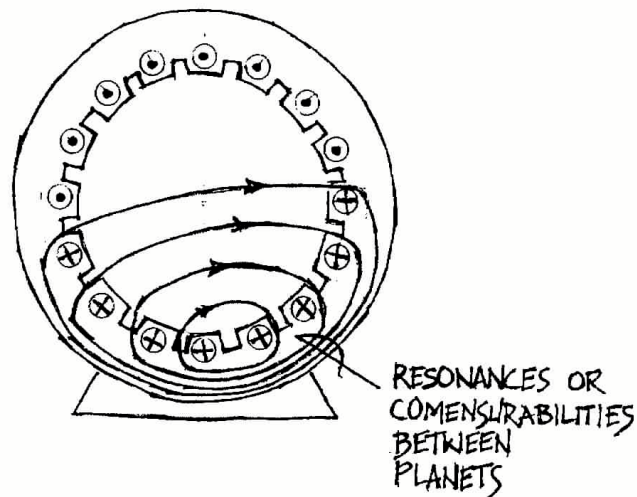
With the rotor, having six windings in the stator, the whole system all together - ten bodies, perhaps even N bodies - behaves as an elaborate 3BP, just as the Szebehely

Equation supposes. The figure suggests a rather uncomfortable presentation of a single planet system, with the central body and moons/rings. This corresponds, however, with the projection of the essentially 2D planar theory so far, into a complicated but not abstract three dimensions. This process will continue more or less until the end of the text, as the theory of Three Bodies is applied to even more systematized 3D systems such as atomic orbitals, which not only have barbell and donut shapes; but have concentric layers of the shapes, all of which implies a powerful central force capable of exacting complex force patterns at a distance.



The Rotor versus the Symmetric Plane

The use of complex numbers previously in this study is natural in the study of rotating dynamical systems, as $e^{i\omega}$ is a natural rotating frequency and the phasors it defines in the frequency domain establish the axis of the planet's rotation, in the complex domain. As illustrated above, the individual planets acts as a salient pole rotor. Presumably this applies mostly to the gas giant planets, which are large enough to have a strong localized gravitational field that can establish an order between rings especially, that is relatively independent of any external influences. Hence, the invariant and symmetric planes establish the field strength of the stator, and a complimentary system of forces exists in the stator for individually independent planet/moon/ring systems as shown here.

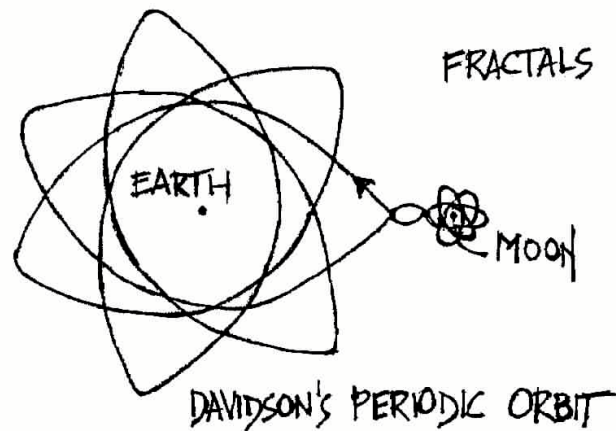


Dynamic Integration between Planetary Systems

The above illustration addresses the existence of dozens of complex resonances between the gas giant planets. These resonances, or comensurabilities, show that the whole body of outer planets has a very intricate sub-organization that is quite beyond the capability of any known theory to explain even superficially. This dialog does not do much better, except that to suggest as the second half of the text proceeds into the complex 3D mechanics of atomic orbitals which are well known; that because the solar system is a planar representation of these same forces, the better known atomic orbital scheme has its analogy in the comensurabilities between the gas giants.

Typically, each gas giant system is one level of the atomic orbital scheme - e.g. the 1s, 2s, 3s, 4s, 5s... - and the resonances between the gas giant planets are structural bonds between these schemes on the orbital level. These exist because of the fractal nature of the forces that are at play in both systems. The method is to apply the 3BP to the better known system, then to overlay this upon the lesser known system and perhaps to use what is well known about the latter to develop insights into the former, and so forth. The more commonalities that can be shown to exist between the disparate realms, the stronger the proof of fractal theory correlations; and the more robust the resulting model.

The best known, but least explained, phenomena in all of this is the fine structure of the ring systems, especially of Saturn. The recent flyby shows ice collecting in some rings; debris in others. This implies a dynamical system that is many orders of magnitude more complex than any known body of theory, which justifies the complexity of this analysis.



Fractals are not necessarily nested

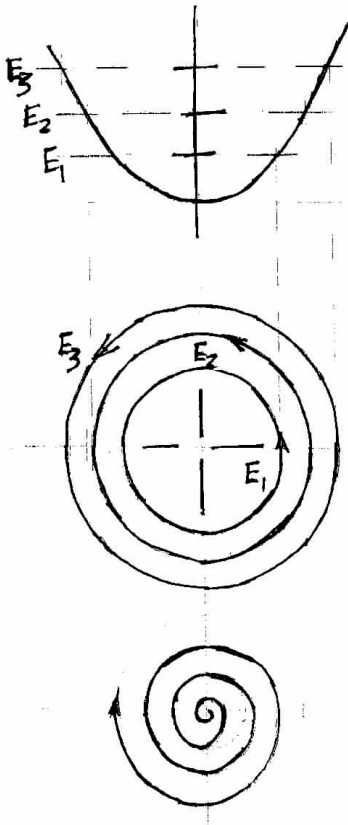
The above illustration of a special periodic orbit in the Earth-moon system found by Davidson shows similar orbits around Earth and the moon linked by a small loop. This supports the notion that fractal patterns may be interlocked in subatomic or quantum systems, but exist in the same plane in systems on the scale of planets. This will henceforth be a recurring idea in the text, and is a useful way to understand the forces at play.

Chaotic Attractors

It is helpful now to study in greater detail the concept of a Lagrange Point being itself a local center or origin of force, as suggested before in this dialogue. Mathematically, regions traversed in phase space are strictly bounded when there is an attractor. In chaotic motion, nearby trajectories in phase space are continually diverging from one another but must eventually return to the attractor. These abstract things are illustrated in the next figure in the general case.

The attractors in such chaotic systems are called strange or chaotic attractors. They are bounded in phase space, as the attractors must fold back into the nearby regions of phase space. They are a kind of transition between fractal levels.

Strange attractors create intricate patterns because the folding and stretching of the trajectories must occur so that no trajectory in phase space intersects any other. Numerical studies of the 3BP show many such patterns, as will be presented in context in later chapters. Mathematically, these periodic orbits are caused by strange attractors.

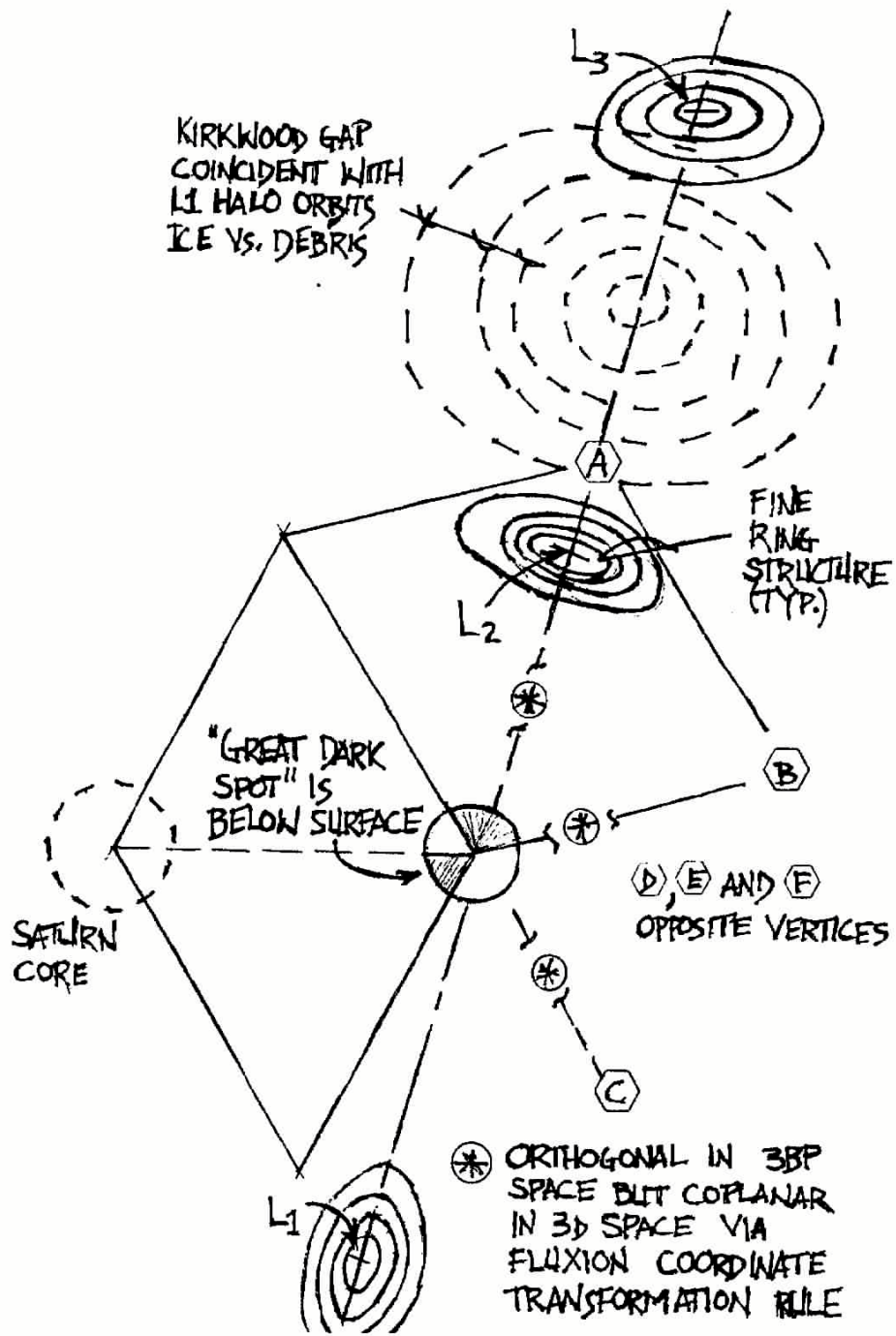


Chaotic Attractors as Fractals

Returning to the more comfortable realm of Three Body motion, consider elliptical motion in a halo orbit around say the L4 Lagrange Point. This would look like the middle figure because the motion is stable (an orbit around one of the colinear Lagrange Points can also be a stable elliptical orbit, but the slightest perturbation or external force applied will cause the orbiting body to quickly spiral away, never to return). In dynamical systems analysis, such points where motion converges for dissipative systems are called attractors. Where beyond the closed ellipses there is what dynamical systems analysts call a limit cycle, but which motion slowly escapes.

The separatrix (9) is another example of this kind of motion, in which two equal masses are the primary bodies (instead of one central body as above). You might say that the figure-8 shaped free return orbit is a stable orbit in the two body system which dissipates into either a higher orbit around both bodies or an orbit around just one of the primaries, when perturbed. This is a more complex type of stability, which implies that the L2 Lagrange

Point is actually a stable equilibrium point when the masses are equal, and not an unstable equilibrium point. This principle is important when considering a system of two masses, which over time evolve so that eventually both masses become equal; at which time a new dynamical equilibrium happens between them and a whole new families of stable and quasi stable orbits are possible, is in the nested systems of atomic orbitals. The following figure attempts to show this idea as it applies to a gas giant with a stable "dark spot" and several systems of complex ring systems. It builds upon the 3D box figure of (13).



Saturn's Ring Systems

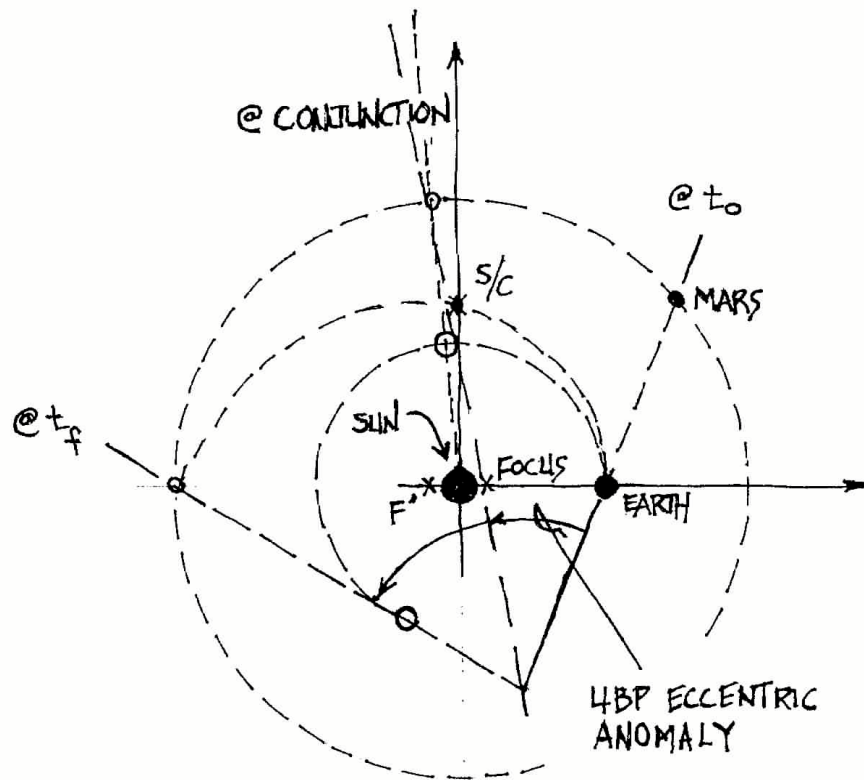
These multiple dimensions are all associated with the same central body - e.g. the main 3BP is the sun-Saturn system, and the orthogonal iterations go sequentially as planets, ring bands (A,B,C,D,E, and F ring systems), and then the fine structure within each ring system. These are consecutive fractal levels that all exist around a single central body. This is quite a complicated force structure to emanate from a single body, which supports the idea of an equally complicated inner structure to the gas giant - here shown with a single core mass and one other mass at the surface manifesting itself as a "dark spot." (Well, Jupiter and Uranus have such spots; this model for Saturn assumes there is a spot, but it is below a surface cloud layer.)

This is just the arrangement of the three types of spherical harmonics associated with the Legendre polynomials and are actually an amplification thereof - whose study will show (31) exactly how the system is structured, and maybe even how it evolves through time.

15 Transitioning Between Coordinate Planes

It has been established that orbits are made up of summed sinusoidal waves. Now there is an hypothesis that two levels of orbits summed this way intersect at an angle of ninety degrees, orthogonal. The problem then becomes how to coordinate these two systems - i.e. the focus of orbits for physical bodies in a dynamically stable system must coincide.

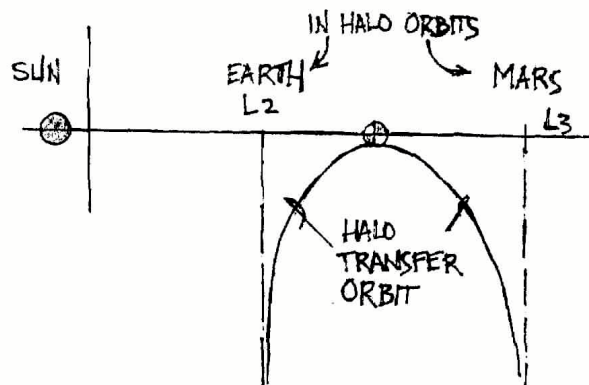
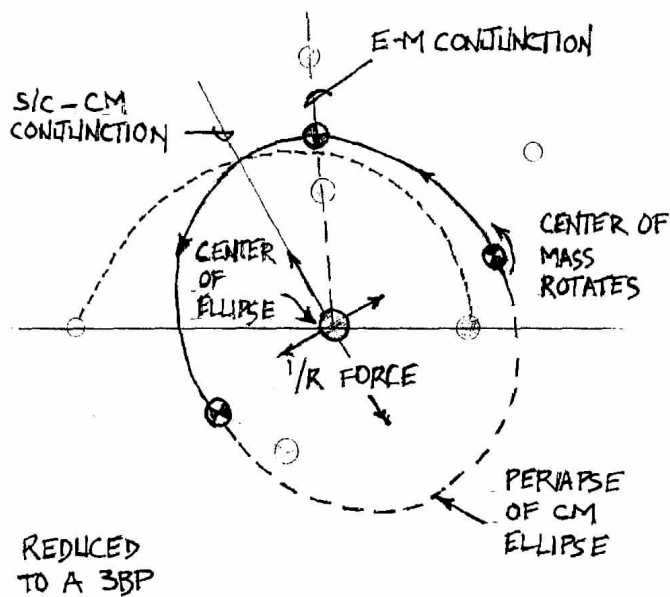
This section has some sketches drawing attention to the fact that the mean anomaly is equal to the angle from the empty focus for small eccentricity orbits. Transitioning from one sinusoidal term to the next for high eccentricity orbits, the actual focus is the empty focus of the previous term, in between there being an increment of inclination added to make this geometrically correct. This inclination term is the primary step function in the symmetric plane, where ellipses remain centered at the center of the ellipse via $1/r$ forces.



An Eccentric Anomaly for the Four Body Problem

This kind of system is amenable to the use of quaternions, and in some cases octurnions.

Now look a little more closely at the figure, to see how the Earth to Mars trajectory can be modeled as a 3BP. The figure below shows how versus the offset angle it is possible to approximate the third body forces on a spacecraft on the heliocentric ellipse to a single quantity, the center of mass of the Earth-Mars bodies. So whenever the position of the spacecraft is known, you also know the forces from Earth and Mars acting upon the spacecraft. Notice that the center of mass is on an elliptical orbit with center at the center of the ellipse, indicative of a $1/r$ force system.

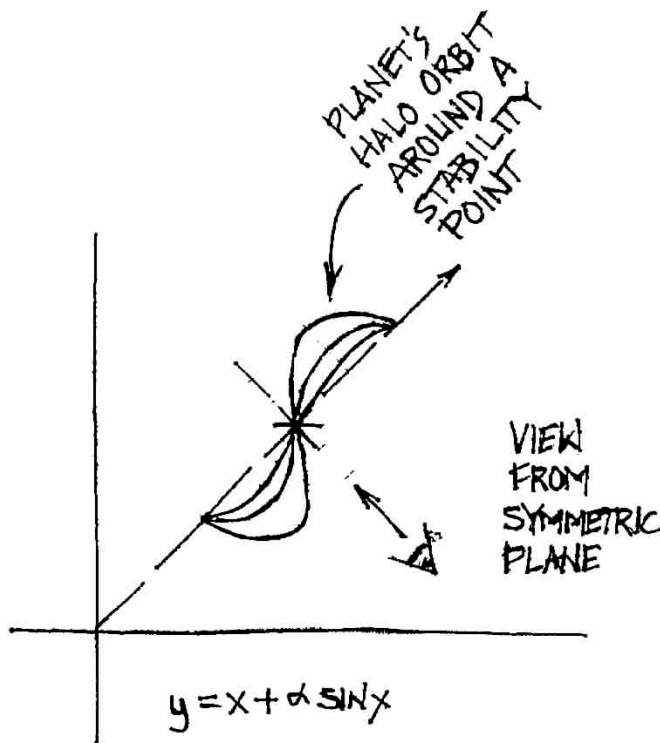


Dynamical Relationships of a Four Body Problem

16 Shock Wave Dynamics

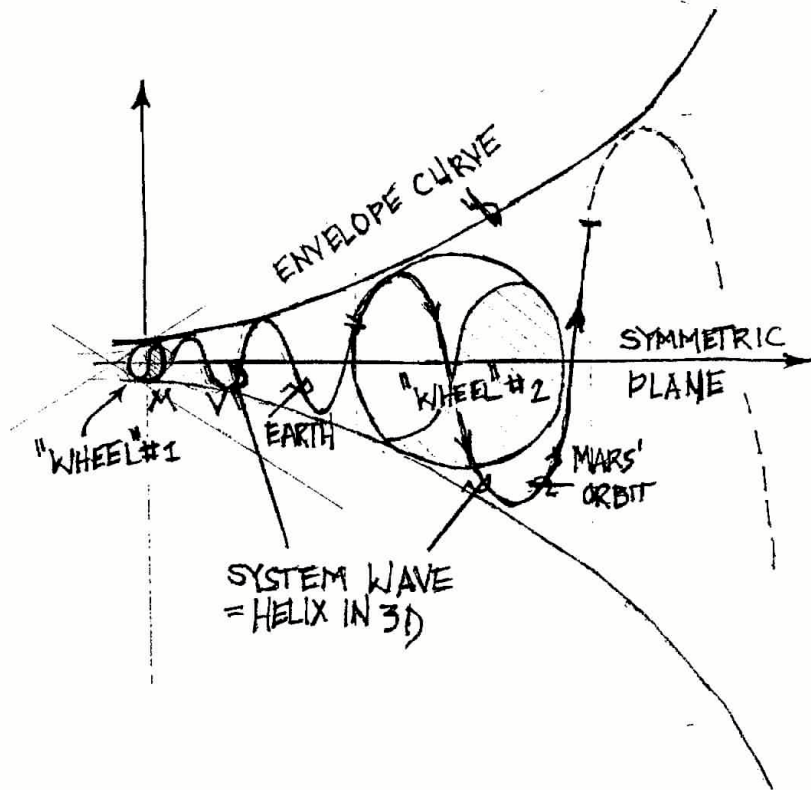
When two wave fronts meet, as in a problem like the Earth Mars trajectory modeled by two boundary value problems, a third wave front is formed called a shock wave.

In multi dimensional space this shock wave must have a beginning and an end - predicated by an exponential envelope and with limits at the inner and outer wheels, like drilling holes at the ends of a stress crack to keep it from continuing to propagate through the whole surface or plane.



Nested Hysteresis Loops

The next illustration shows how, versus the system wave diagram, the planets form a continuum of a sinusoidal wave vis-à-vis an envelope curve.



Symmetric vs. Invariant Plane and Planetary Motion

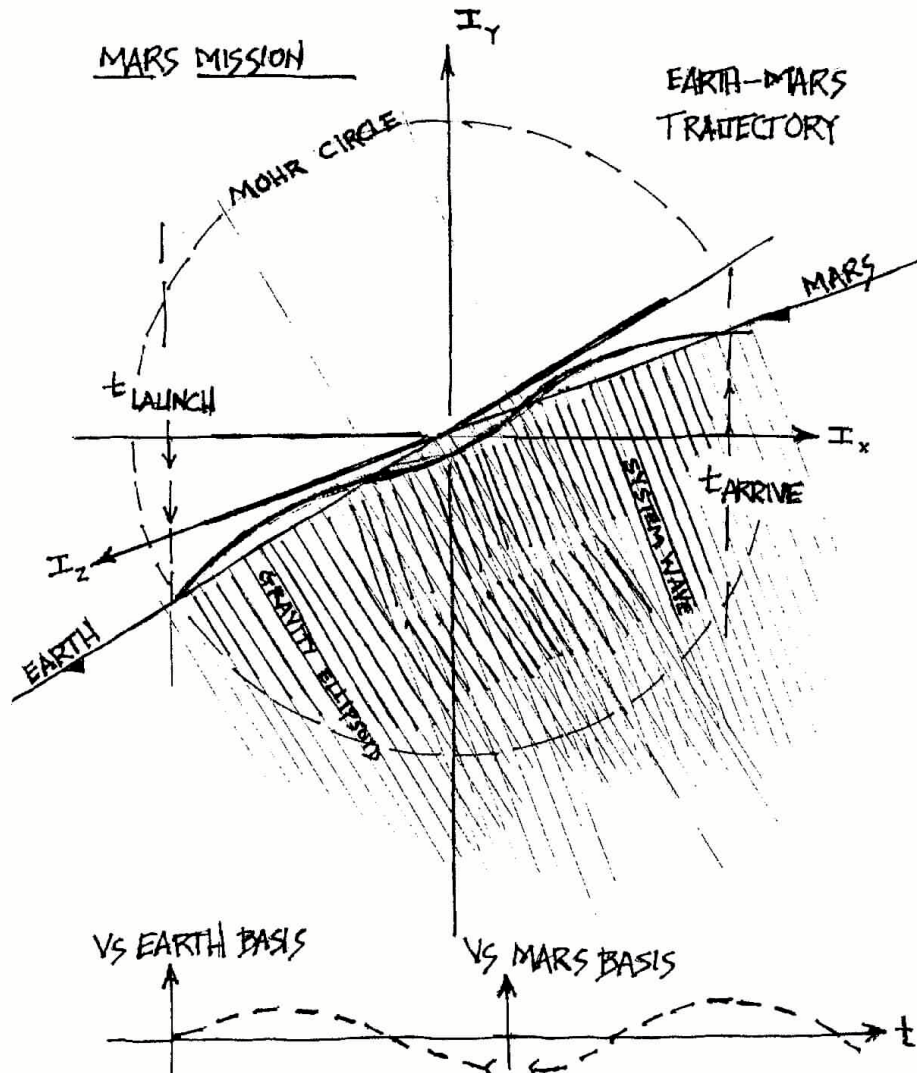
The planets form a continuum of a sinusoidal curve, versus the envelope curve, which is consistent with the system wave hypothesis. This kind of a phenomena is exhibited on the atomic level (recall that the equation for gravity is the same as the equation for electrostatics, except for the constant of proportionality).

Electrostatic potential in the vicinity of a nucleus is modeled by the potential function

$$U(r) = -\frac{k}{r} e^{(-r/a)}, k > 0, a > 0$$

called the screened Coloumb potential because it falls off with distance more rapidly than $1/r$ thus taking into account the partial cancellation or screening of the nuclear charge by the atomic electrons. It is worth noting here that, under certain conditions, there can exist bound orbits for which the total energy is positive.

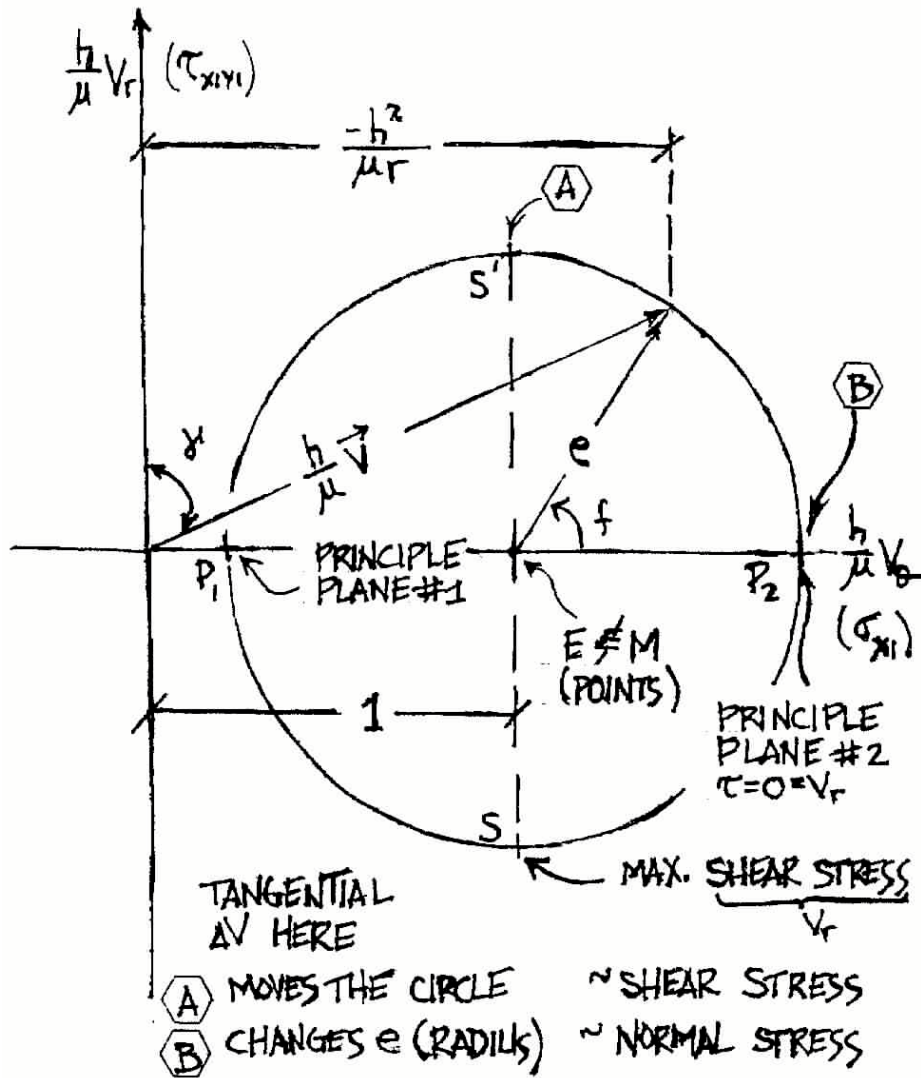
The next illustration is a study of the Earth to Mars trajectory that lends itself to the Mohr stress/strain analogy of the following figure.



Relationships between the various Conjunctions and the Time Lines

The following figure draws on the similarities between the velocity Hodograph or phase diagram for an interplanetary trajectory and the stress-strain diagram called Mohr's Circle used in the analysis of solid objects under load. The latter has two principal planes as

noted, and stress is correlated to velocity in the radial direction; strain to velocity in the tangential direction.



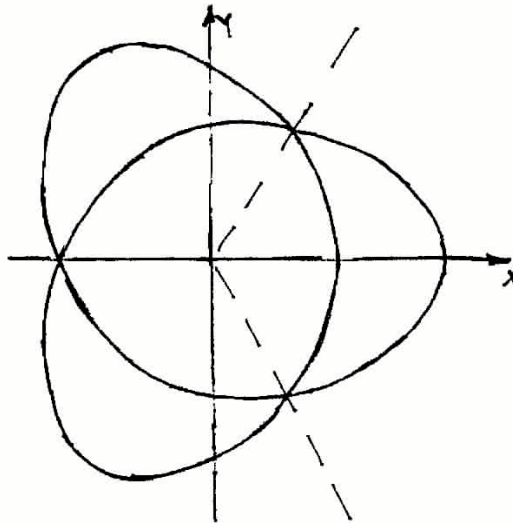
The Engineering Mechanics of Interplanetary Trajectories

17 Quaternion Model for Eccentricity Accumulation

This speculative paper extrapolates the idea that hysteresis is reminiscent of accumulation phenomena (a fundamental concept in solid state physics theory, to be covered later) because a dynamical system behaves differently when it is increasing than when it is decreasing.

Solar system data shows a repeating pattern, like in atomic orbitals (e.g. 1s, 2s, 3s,...) - fractal theory at work.

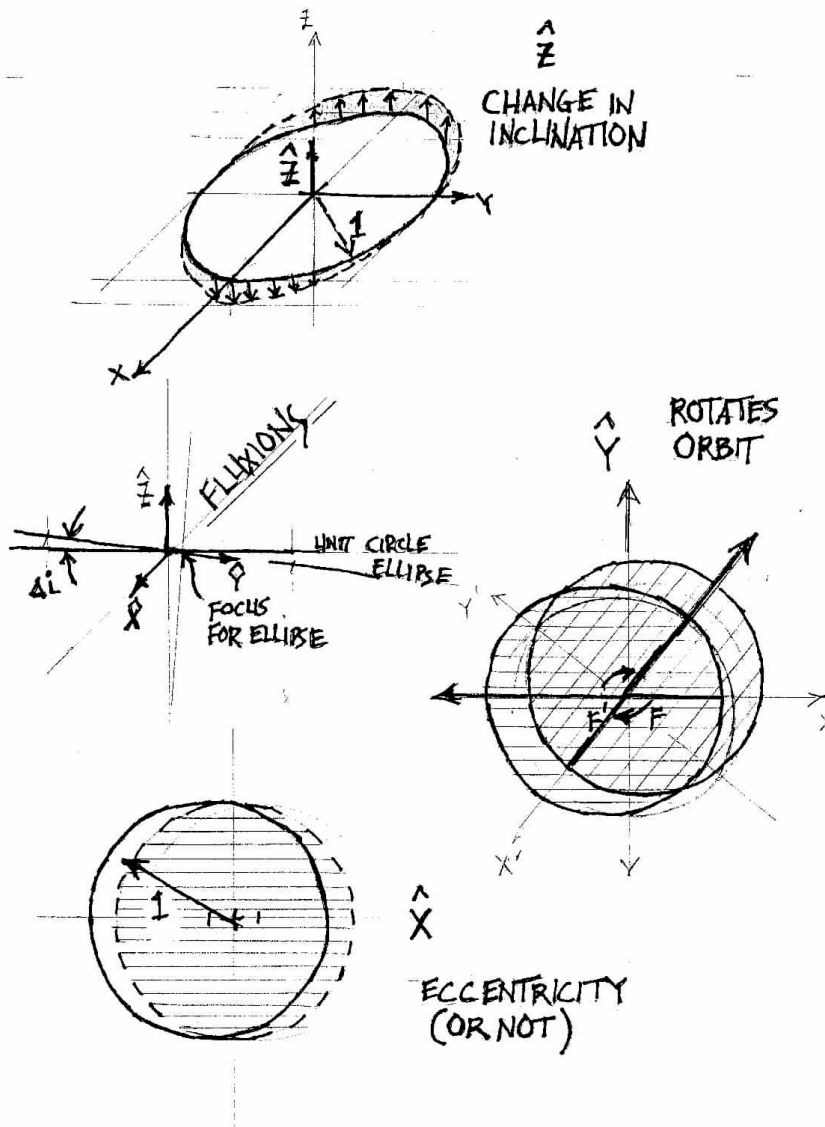
Within each level, orientation of e-increments presumably changes. Many variations are possible. A "Wankel Engine" is likely a stable 3BP triangle of equilateral points up to perhaps a twelve cylinder rotary aircraft engine (developed later). Following is a periodic orbit of the 3BP that is characteristic of the rotating triangular shaped pistons in the Wankel Engine.



A Periodic Orbit in the Three Body Problem

A mathematical shorthand representing each transition from circle-to-low-eccentricity-ellipse is helpful, showing how transformations along each coordinate axis affect various

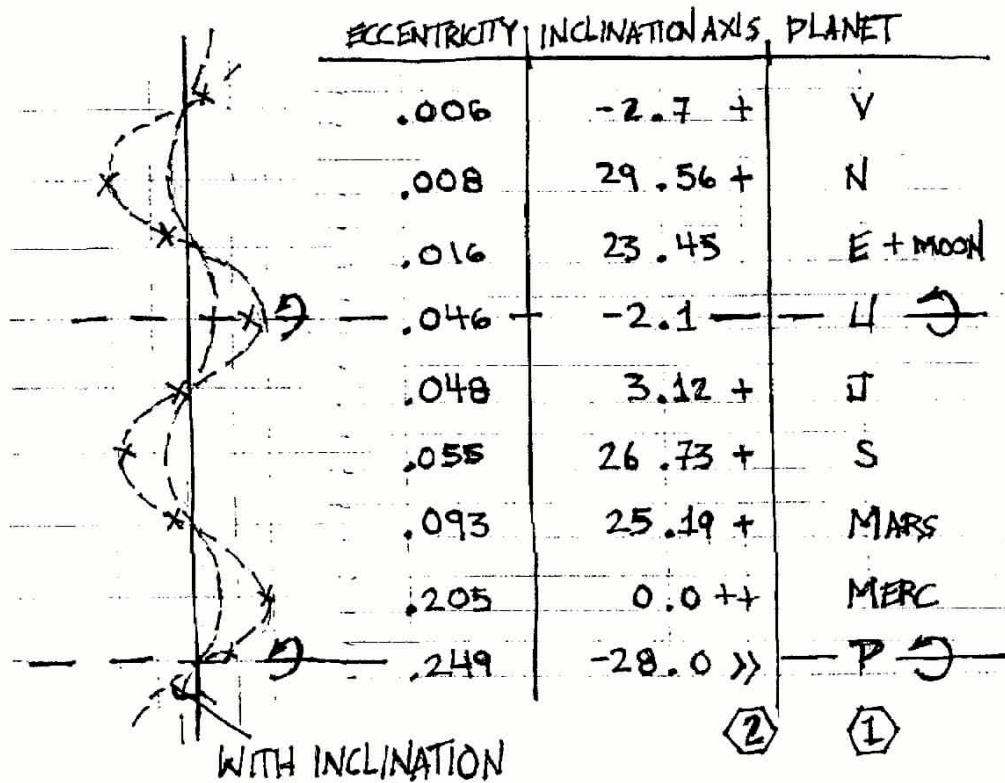
aspects of the ellipse - e.g. the orbital parameters. Following is an illustration suggesting how the three coordinate axes might be interpreted.



The Fluxion Coordinate System

The general idea is to develop a Cartesian type coordinate system so that phasors along each of the orthogonal axes cause a unique change in the orbit and so that, all together, they encompass all of the nine orbital elements.

Another requirement for this "Fluxion Coordinate System" (Fluxion is the name given to elements of the Calculus by Newton, and seems an apt word to use in this particular application) is for there to be some kind of nesting mechanism, allowing sequential representation of fractal levels. The following figure hints of this kind of nesting in the orbital elements of the planets, which suggests that the scheme shown above might work.

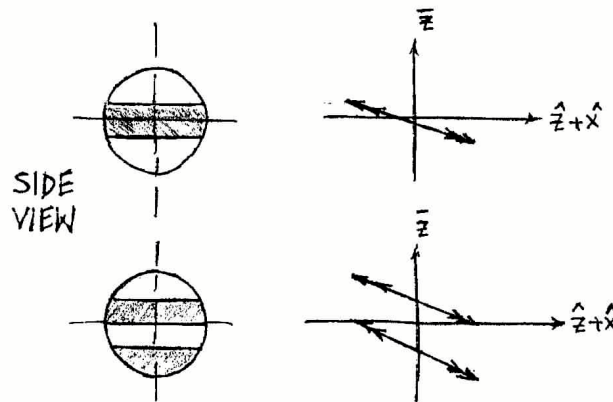


① MAX SUSTAINABLE INCLINATION = 30°
STARTS OVER e 0.5°, 1.0°, 2.0°

② INCLINATION TO ECLIPTIC

Orbital Elements versus the Fluxion Coordinate System

The next requirement for the Fluxion system is that it must exhibit the kind of structure that underlies the geodesy of Earth as revealed by fourier series modeling. Sturm-Liouville theory says that the spherical harmonics are eigenfunctions (which are mutually perpendicular or orthogonal functions) that are an independent basis for the gravitational potential model. Since these terms also correlate to the eccentricity equation (1) and each term is an eigenfunction; they are all orthogonal, so the form a basis for modeling the Earth's gravitational field. Consider the simple case of the zonal harmonics, and how the represent graphically the higher order terms of the eccentricity equation.



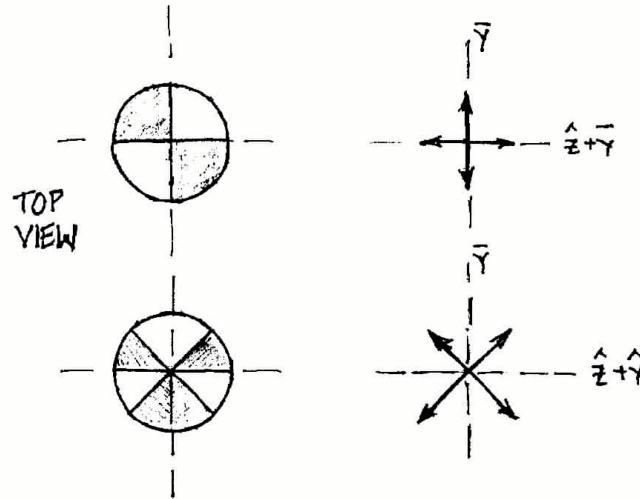
Zonal Harmonics and phasors in Fluxion Space

The general idea is to orient these phasors so that they are aligned parallel to one of three principal axes (essentially the organization of a gravity ellipsoid, if one could be derived analytically). This notion is posed by MacCulloch's Method (Vallado, 488)

$$U = \frac{Gm_{\oplus}}{r} + \frac{G}{2r^3}(A + B + C - 3I)$$

where the first term is the gravity potential, the second is the two body potential, and A, B, and C are the moment of inertia about the three principal axes; and I is the polar moment of inertia. This is called MacCulloch's Formula.

The sectoral harmonics are not much harder to conceptualize because they are symmetric about the polar axis, as indicated in the next figure. Here the phasors are seen to form the boundary lines between the different zones, which is exactly the mathematical significance of the Lagrange polynomials.



Sectoral harmonics and their Fluxion Equivalent

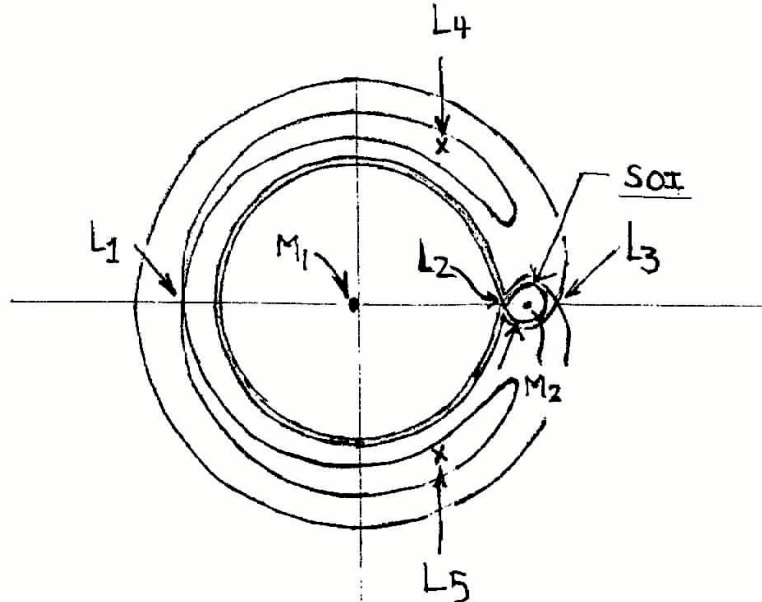
The tesseral harmonics are more complex, and are essentially a combination of the above two methods, displacing a phasor and rotating it about the polar axis, to form the lines between the tesseral segments.

18 Intergalactic Envelope Curve

The inner and outer "wheels" (or envelope curves, fitting the apoapse of the comet orbits around the sun and Jupiter respectively) are more than the pivots for the symmetric vs. invariant planes (two charge plates that electrolyze water, the N Body Problem, into the N-1Body Problem) but the small loop of two free return 3BP's, both with our sun as the second, small body. Refer to (16) for the referenced diagram.

These are permanent structures that maintain the dynamic balance of the solar system as it evolves from cosmic cloud to discrete planets. Presumably the paths of comets are along neighboring optimal paths to these interstellar "gravity strings." A spaceship should be configured to match their cross section for high speed travel, perhaps greater than c .

The "inner wheel" may be the anomaly within the sun that causes the precession of Mercury's perihelion. The "outer wheel" may be the mysterious Red Spot on Jupiter. These phenomena are situated inside their respective bodies (Jupiter is a gas giant) because their gravity is at a point mass otherwise (recall that concentric shells of uniform density can be represented at some radial distance away as a point mass).

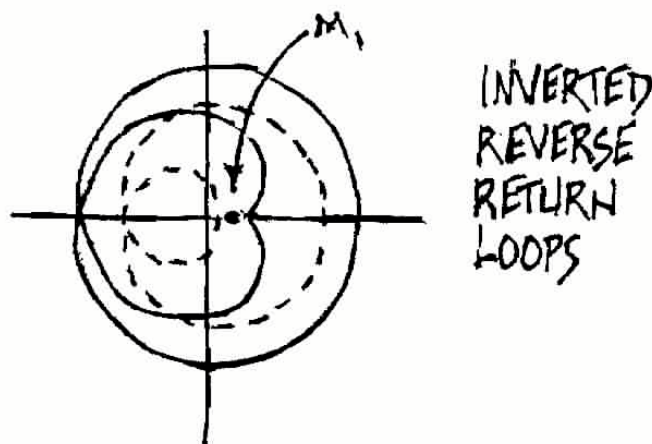


IV. A Unified Field Theory

The goal of a Unified Field Theory (UFT) is to derive a set of equations that includes all four fundamental forces of nature - weak and strong gravity, weak and strong nuclear forces (i.e. electromagnetism - electricity and magnetism). The 3BP does just that because the equations of motion for electrostatics and gravity are both inverse squared forces, differing only by a constant of proportionality. Thus, the 3BP is the UFT, with the only dubious thing being how to differentiate weak and strong gravity. (ONE)

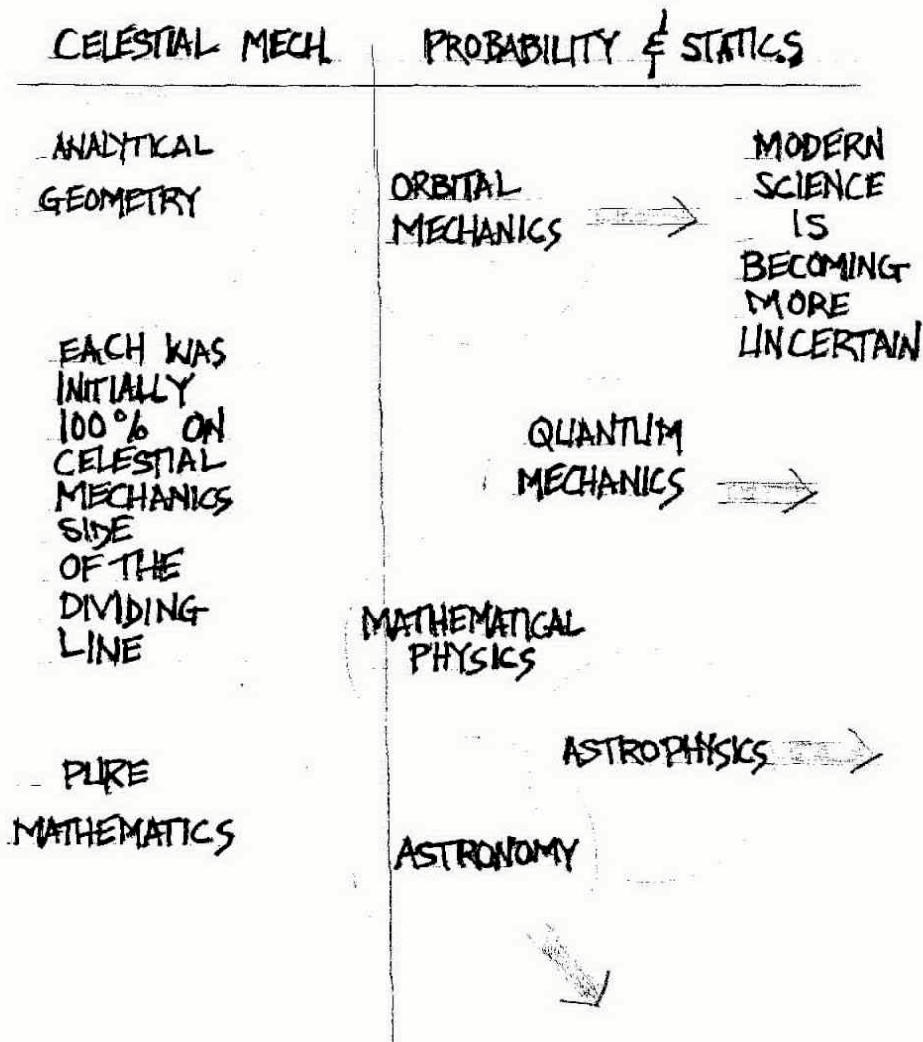
The intention of this section is to show that the 3BP is even more ubiquitous than a UFT. Having long since accepted the applicability of the 3BP in so disparate realms - astrophysics and atomic physics - it's logical that, according to fractal theory at least, this 3BP scheme applies to every discipline in between these extremes. The original Bohr model of the atom (one electron orbiting a proton) was taken straight from Celestial Mechanics, as the Two Body Problem. It is now possible to do the same with the 3BP, starting from the very beginning.

The motivation for this research is to break the gridlock that is pervasive throughout the natural sciences. (11) Virtually every fundamental discipline is now strictly based upon statistical methods - from astronomy to solid state physics, from quantum mechanics to quantum chemistry. What is not statistics or probability theory is modeled numerically on powerful computers using analytical approximations.



Statistics and numerical approximation are good tools but they have limitations. They give no insights into actual physical processes and in their generality make it impossible to perceive the finer aspects of physical and dynamical systems.

This research endeavors to qualify and quantify (FIVE) the vagaries of statistical methods by suggesting more detailed methods for these things - e.g. a 3BP model instead of the Bohr 3BP. Atomic bonds are no longer a vague concept, but a "free return" orbital between two nuclei positioned as the primary bodies in a 3BP. (22) An even more interesting model is proposed for the semiconductor transistor device. (21)



The place of Celestial Mechanics in modern science

Models are very powerful research tools. The 3BP is a well known system, with many applications clearly evident in nature. A model of the transistor, say, that shows close parallels between the statistical model and the 3BP has the potential of giving solid state physicists a better understanding of their field. This, in turn, opens a virtual bonanza of new ideas that promise to reveal the physical mechanisms that happen in the transistor. Electrons and "holes" don't migrate across a "conduction band," particles on specific trajectories traverse a region analogous to the N Body Problem where the equations of motion reduce to a fluid - i.e. the so called conduction band. A good model can show how the methods used to get spacecraft from Earth to Mars efficiently can be used to get an electron from the emitter to the collector in a transistor. Likewise for the basic processes in quantum physics and quantum chemistry.

19 A Model for Charged Particles

It may not be inconsequential that the computer simulation of the Earth to Mars trajectory has some subroutines that approximate the behavior of positive and negative charged particles (recall the equations of motion for gravity and electrostatics are identical except for a constant of proportionality). The Earth "capture" of the spacecraft is very easy to optimize, behaving like two oppositely charged particles. The Mars capture is exceedingly difficult (in theory as well as in practice), and the problem behaves as two like charged particles.

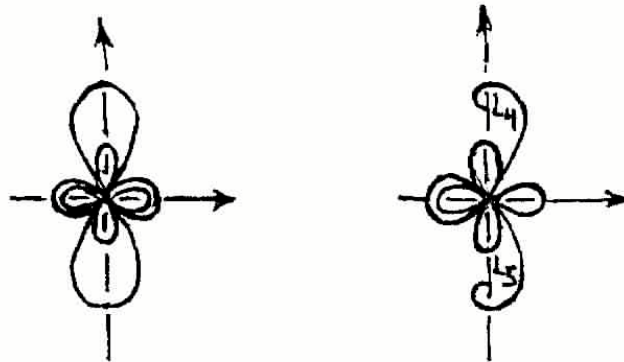
First consider the Earth "capture" routine (8), reverse integrating from conjunction to Earth. Earth is a large, massive body - like a positron compared to a tiny electron, the spacecraft. It was simple to create a dependable and convergent algorithm to target an Earth parking orbit from conjunction. Earth has a large sphere of influence (SOI) that is easy to target because Earth and the spacecraft track a similar trajectory, thus there is a wide "window of opportunity" that permits large step sizes, every one of which is assured a solution.

The Mars capture routine (9), a direct integration from conjunction to Mars, is exceedingly difficult by comparison to the Earth "capture" routine. Mars is much smaller than Earth, and their relative behavior in the numerical model is similar to the behavior of two like charged particles. It is hard to target Mars' SOI from conjunction, the "window of opportunity" being small because their orbital trajectories and velocities are so dissimilar. In fact, it was literally not possible to create an algorithm to solve the problem directly. The routine stops the integration near SOI, then literally "skewers" Mars - fixing the conditions until a successful trajectory to a parking orbit around Mars is achieved, then integrating back through time from this point - to SOI, then conjunction.

The difficulty in solving the conjunction-to-Mars problem as likened (9) to a pedestrian standing at a railroad crossing, waiting for the locomotive to approach - then making a dramatic last minute maneuver to get on board the rapidly moving train. This situation is predicated by their relative velocities as well as the geometry of how the orbits intersect. There is literally just one solution - modeling the approach on a sun/Mars free return loop - out of literally thousands of "good possibilities." This is just the kind of behavior attributed to two similarly charged particles.

The real life solution to this problem is actually more difficult than posed in the algorithm because the simulation is a 2D one in which all bodies move in the same plane. In

reality, Earth and Mars orbit in planes inclined at about 3 degrees, so the spacecraft approaches Mars in one orbital plane, and the sun/Mars free return orbit is in another plane - e.g. the Mars orbital plane. So the last second maneuver at the L2 point on Mars' SOI includes a slight course correction as well as a plane change (although ideally this plane change would have been done before, when the Earth/Mars orbital planes intersected so that the spacecraft approaches Mars in the same plane as Mars' orbit). This further accentuates just how hard it is to get a spacecraft into a safe orbit around Mars, something all the space agencies have learned the hard way, having collectively only a 25% success rate in all their Mars missions.



20 Solid State (Astro)Physics

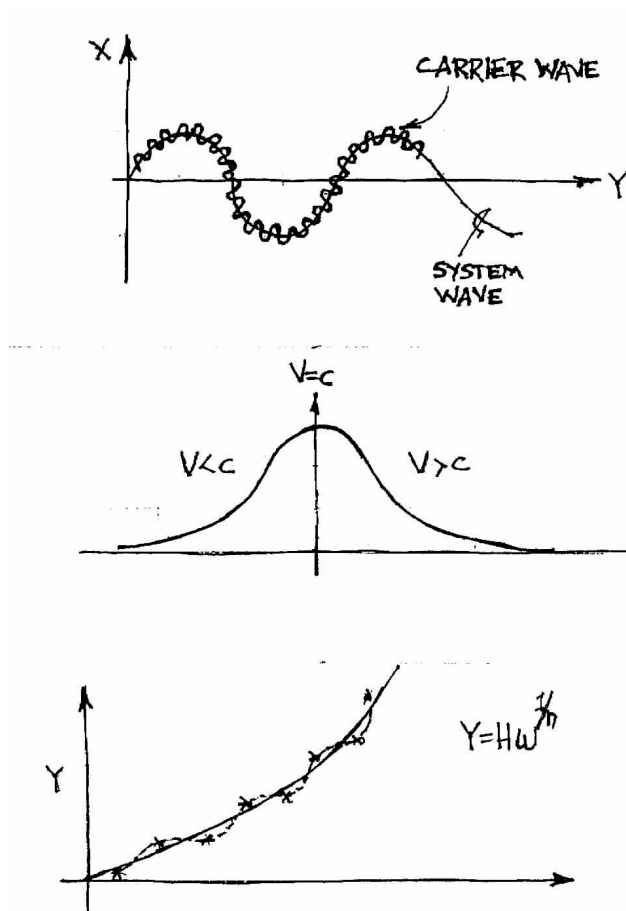
If you take the "system wave" and rotate it and resize it, you can form a set of repeating patterns that fit very nicely the Schrodinger Electron Cloud Plot solutions of quantum mechanics. This solution is actually more than that, because it shows the Cloud Plots to be several nested sequences of a Fractal Theory - i.e. similar patterns repeating at subsequent levels of magnitude.

This becomes more than a curiosity when you do a simple analysis of all the comet's paths as they move in the vicinity of Jupiter (the so-called "Jupiter's family" of over thirty small comets). Assuming that the orbit of Jupiter was at the edge of one of the Cloud Plots, which can be generated using the "system curve" as 2d sections of the 3D plots (e.g. spheres, barbells, and torus shapes), if you were to extend them slightly above and below this 2D section and sprinkle the outer shell with powder, you would get exactly the silhouettes of the comets paths versus Jupiter.

The comets, through their trajectories, have traced the patterns of 3D forms that, at Jupiter's orbit, are barbell shaped. Thus, the "galactic atom" theory goes from being an interesting curiosity like Bode's Law, to being a possibility; if not a probability. The logical progression of these hypotheses is that the pattern of electrons orbiting an atom is repeated at successive levels, up to the level of being represented on the scale of our solar system. There should, by Fractal Theory, be some intermediate level(s). It is possible to show the existence of a carrier wave to the "system wave," a small sinusoidal pattern superimposed on the much larger system wave. This small pattern has a direct correlation in the only aspect of the planets' motion not yet fit in the theory: the rate of rotation.

All of this is mostly just a geometric curiosity shown to fit a complex system. It's historically correct, conceptually reasonable, and mathematically correct, but there is no fundamental basis. In light of the other papers here, it is possible to propose that the system wave is gravitational, and the smaller carrier wave is electromagnetic in nature. (29) They may have the same frequency, having a common origin in the sun; which would make the larger one travel much, much faster - as we know gravity does. It's also reasonable that the smaller wave is electromagnetic, having its subtle effect on the rotation of planets because of the magnetic core of the bodies; i.e. acting like a magneto to induce rotation as implied in the paper proposing a cause for the so called seasonal variation of the Earth's rate of rotation. (29)

The interesting part is that the carrier wave is not of constant magnitude; it has a slowly increasing magnitude. This implies that light waves do not travel unchanged through space, especially since space in the vicinity of a solar system; but is quite noticeably altered by it. This, in turn, implies that the distances we compute to nearby stars are not as far away as calculated.

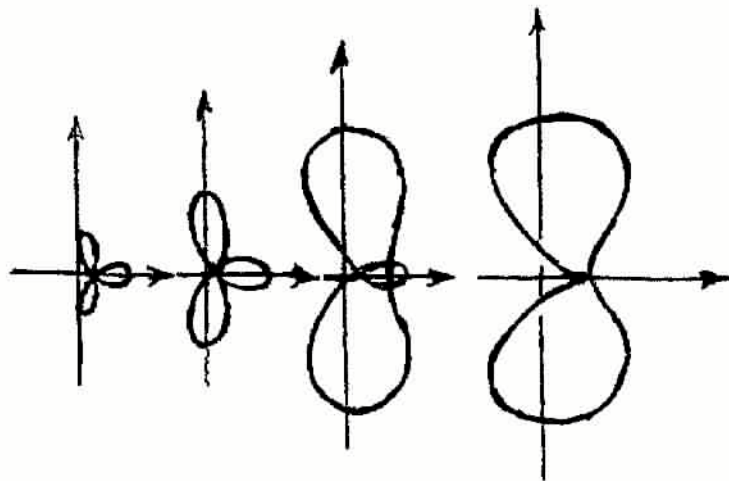


Consider the EM carrier wave (i.e. the variations from the symmetric plane in (2)) which has as its x-axis the gravity or system wave. The distance traveled by the carrier wave is greater than the shortest linear distance between two points because it's traveling along a 3D helix and the physical distance is the straight line down the center of this helix. It's like accepting at face value that electrons travel at the speed of light through a conducting metal (21), whereas electrons do not actually go directly from one point to the next but have a sometimes widely circuitous route, going around molecules, anomalies, and so forth - so that

an individual electron on average travels at the speed of light c , but may travel quite a lot faster following a Gaussian type distribution.

Assume that the EM waves have a preference to use gravity waves as their axis of propagation (refer to the analogy for the Szebehely graph to the 3BP). This means that gravity waves can be silhouetted by EM waves (like the Northern Lights and the Earth's magnetic field) using a giant oscilloscope the size of a football field (because gravity waves are so large, any lesser distance and the gravity wave would just look like a straight line). Say you have a 250 foot long shaft of a large power planet, driving a turbine. If a gravity wave on that order of magnitude exists, then it could theoretically be detected, and perhaps lessened in intensity by virtue of the EM carrier wave (which is presumably easier to emulate). If the energy to create and sustain this carrier wave is less than the energy saved by the power plant by operating in a less intense gravity field, then it is a viable engineering application. (It is possible that gravity may be simulated using the mass analogy to charged particles (19).)

On a larger scale, say you set up this giant gravity wave oscilloscope (perhaps all you need is a huge sheet of mylar in a low Earth orbit) at Cape Kennedy, and isolate a gravity wave existing at the shuttle launch site. Configuring the shuttle flight path to follow this lazy spiral away from Earth's surface might save fuel. If the shuttle could mimic the EM carrier wave this might save even more fuel, the shuttle thereby following the gravity wave into orbit like a mono rail train. Eventually the system might be engineered to avoid the force of gravity all together, or at least a larger percentage thereof.



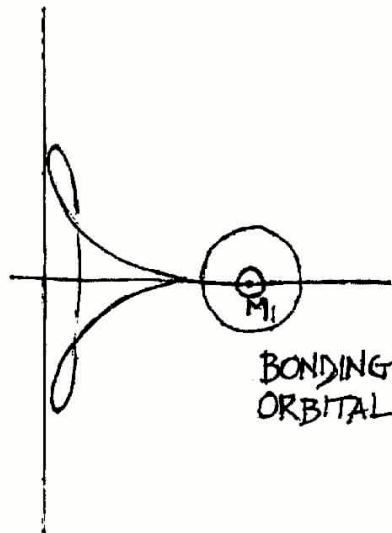
In deep space beyond SOI, presumably gravity waves intertwine in increments of three - like the 3BP with two primaries and one body at L4/L5 in the collapsed rotating system

- and a spacecraft designed to imitate this spacing could ride the gravity monorail at high velocities up to, perhaps beyond, the speed of light . . . after which point the whole pattern repeats.

Presumably there is some radial distance from the center of the Earth at which point the gravity waves begin to coalesce. In the case of the solar system held by the sun's gravity, it may seem somewhat paradoxical that the outer planets are far more massive than the inner planets. Logically, the sun's gravity being stronger closer to it, you'd think there would be much more mass closer to it. This makes sense, however, if the gravity flux becomes more organized the farther you get from the sun, traversing a sort of event horizon at the asteroids. It helps to quantify this idea, and I have created a value that reflects the total rotational (about the axis of rotation) and kinetic (moving around the sun) energy of the planets. It increases rapidly at a uniform rate, symptomatic of the envelope curve for the motion versus the system wave. (3)

In the context of gravity, consider again the model of a massive body using concentric spheres of uniform density and thickness. At some point in the distance, the body acts as a point mass. Yet, within any given sphere(s) all the forces cancel, so the exact center of a sphere constructed in this matter is an "all forces nullification point: - i.e. there is no gravity at all.

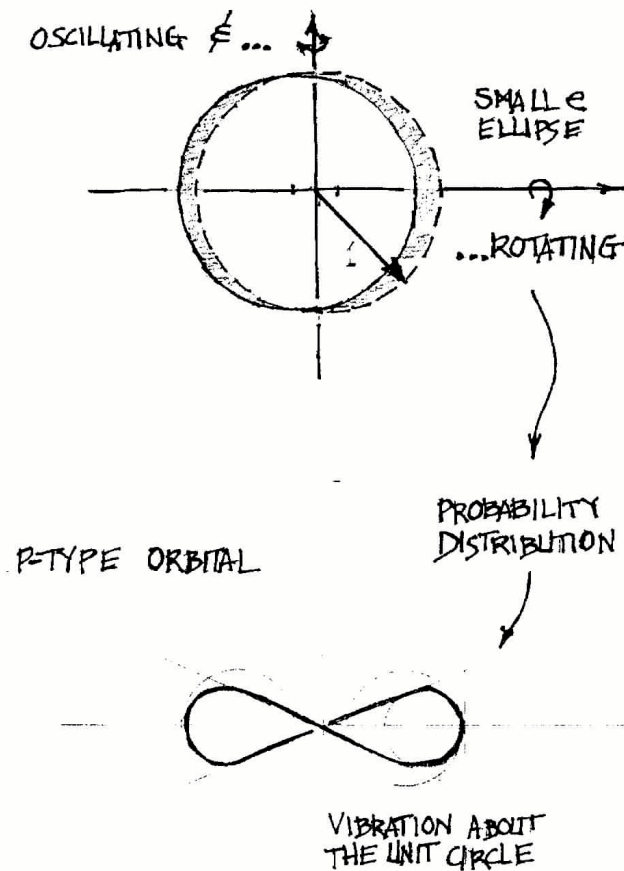
This implies a measured transition, which in turn suggests - when waves are associated with the system, as a Fourier Series of discrete energy levels - that all the waves are harmonics of a single fundamental frequency (e.g. the "system wave" (4) suggested this idea), and that there are specific radial distances at which they combine.



Thus, a shuttle aligned to a gravity wave at Earth surface might, as it goes upward, be accelerated at several different rates, as its gravity "rail" becomes stronger and more defined. Similarly, travel in the spherical plane perpendicular to the shuttle's path, at the transition state where new waves coalesce into the whole, it might be possible to travel at a very efficient rate - e.g. a "jet stream" type of phenomena. A similar phenomena might exist in the oceans, a type of inversion layer near the surface.

21 The Conduction Band

One of the most unusual phenomena in any physical science is the concept of a "conduction band" in modeling electrical current flow in conductors. There is no theoretical basis for the process in electromagnetic theory or in solid state physics (i.e. the study of the transistor junction). The latter is literally 100% statistics, yet it has a very well defined notion of this "conduction band" in which free electrons from all the metal molecules have no formal association with a specific molecule (as is the case in all other known materials as modeled by all other physical sciences), but instead occupy a theoretical realm in which all of the electrons can move free and effortlessly through the material at close to their maximum possible speed - the speed of light.



This "conduction band" is endowed with other very special properties, which are not supported by solid state physics theory in the least. Relativity Theory says that any object approaching the speed of light experiences an exponentially increasing mass - even at half the speed of light (c) the mass has increased many fold, much less at near to c itself. Somehow electrons in this "conduction band" can violate Relativity in its entirety.

The only theory that justifies this violation of every precept of Relativity by electrons is the N Body Problem of Celestial Mechanics. Relativity was actually derived long before Einstein (11) assuming an aether (instead of an electromagnetic field - the old particle mass duality of physics). Einstein's Relativity says motion is impossible near, much less beyond, c , because the object has that point reached an infinite mass value. Poincare's Relativity says motion at c behaves like incompressible fluid flow - i.e. the equations of motion for the N Body Problem - and makes no constraint for motion beyond c , except that thereafter motion is within an incompressible fluid. The universal acceptance of the "conduction band" model shows that Celestial Mechanics is the better science because this idea does not violate any of its laws, much less all of them.

The other papers in this section offer analogies from other perspectives, showing how the 3BP and other principles of Celestial Mechanics closely approximate the behavior of subatomic particles - in molecular bonds, atomic orbitals, and in the overall subatomic force structure. Taken together, they make it seem plausible that conductors have some kind of molecular structure that simulates the forces of N bodies upon each small body, giving the "conduction band" idea the feel of incompressible fluid flow at the atomic level. It is possible to show many more parallels between astrophysics and solid state physics, and strive to suggest a discrete - rather than a statistical - model of the transistor junction.

Observe that, once again, the N Body Problem from a large perspective reduces to the 3BP. This implies that once c is reached, space is structured like a 3BP again - and when $2c$ is reached the N Body Problem is solved, etc.. This is consistent with the $1/r$ and $1/r^2$ force criteria established earlier. (ONE) The nature of the fundamental forces changes with each new level, although the physical aspect of bodies in each space is unchanged level to level. Each level is bounded by two planes, one at the minimum (the invariant plane) and the other at the maximum (the symmetric plane). At the next higher level, the minimum is the maximum from the lower level - and conversely for the next lower level.

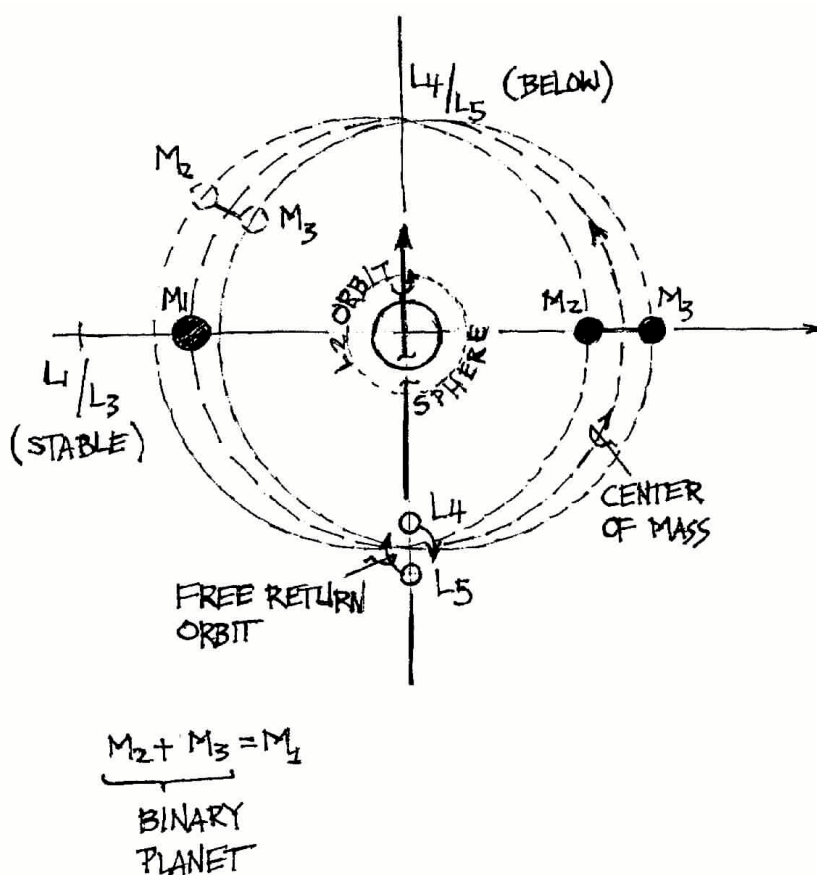
MECHANICAL TRANSLATION	ELECTRICAL	REMARKS
FORCE / THRUST, ΔV	POTENTIAL, V^+	{ EARTH ~ P-TYPE MARS ~ N-TYPE
SPACECRAFT, s/k	ELECTRON, e^-	
HOHMANN TRANSFER	JUNCTION DIODE	VOLTAGE (THRUST) APPLIED @ EACH
EARTH	EMITTER	
MARS	COLLECTOR	
SUN	BASE	MOSFET CONNECTS @ LAGRANGE POINTS
s/k @ RETURN	HOLES	
VELOCITY	CURRENT FLOW	

Otherwise, the structure of junction diodes is equivalent to an electron going from a stable orbit in one material across a uniformly charged region to a stable orbit in a second material. Historically, transistors have been optimized by having a small forward bias in three places: across the emitter, the base, and at the collector. This is exactly analogous to the optimum trajectory I have found: a thrust from the initial Earth parking orbit, one at midpoint in the "base" region, and another at the final Mars parking orbit. The electronic model for the junction, as embodied in the Ebers-Moll equations is exactly analogous to this. I have even found what act like Lagrange Points in the more advanced transistors, the BJT and Mosfet. The advances in solid state physics have followed exactly the same lines as those in celestial mechanics.

The bonus in this kind of investigation would not just be a better transistor, or a confirmation of the Chaos Theory in finding another level of the repeating body of theory. The bonus would be in finding a discrete analogy to statistics. It would put some physical significance to all the abstract statistical methods that have been developed. This would allow many fields to be better understood, as statistical processes were endowed with some physical significance.

22 Bonding Orbitals

The atomic bonds between atoms or molecules are the transitions between fractal levels. These bonds are assumed to be made up of electrons on "free return" paths such that the particles linked are the two primaries in a 3BP and the bonding orbital is the zero velocity curve through the L2 point. Fission need only destabilize this point; fusion need only put a particle in a halo orbit around it.

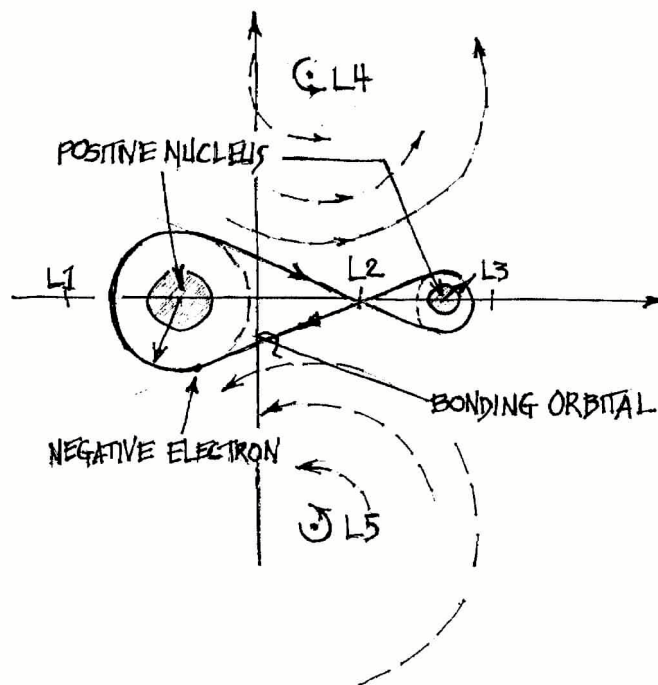


Orbitals Above Higher Energy S Type Bonds

The source of gravitational field force is mass and the source of electrical field force is electric charge. Otherwise they are exactly the same mathematically:

$$F = G \frac{m_1 m_2}{r^2} \quad \text{and} \quad F = k \frac{q_1 q_2}{r^2}$$

This means that there exists a Three Body Problem on the subatomic level, viz a proton, electron and a third body ~ a proton, if the study is to look at intermolecular bonding. This is similar to the famous Copenhagen problem of celestial mechanics, wherein both massive bodies have unit mass and unit separation. The following illustration shows two nuclei of the same positive charge, with one perhaps having a large mass because of nucleons.

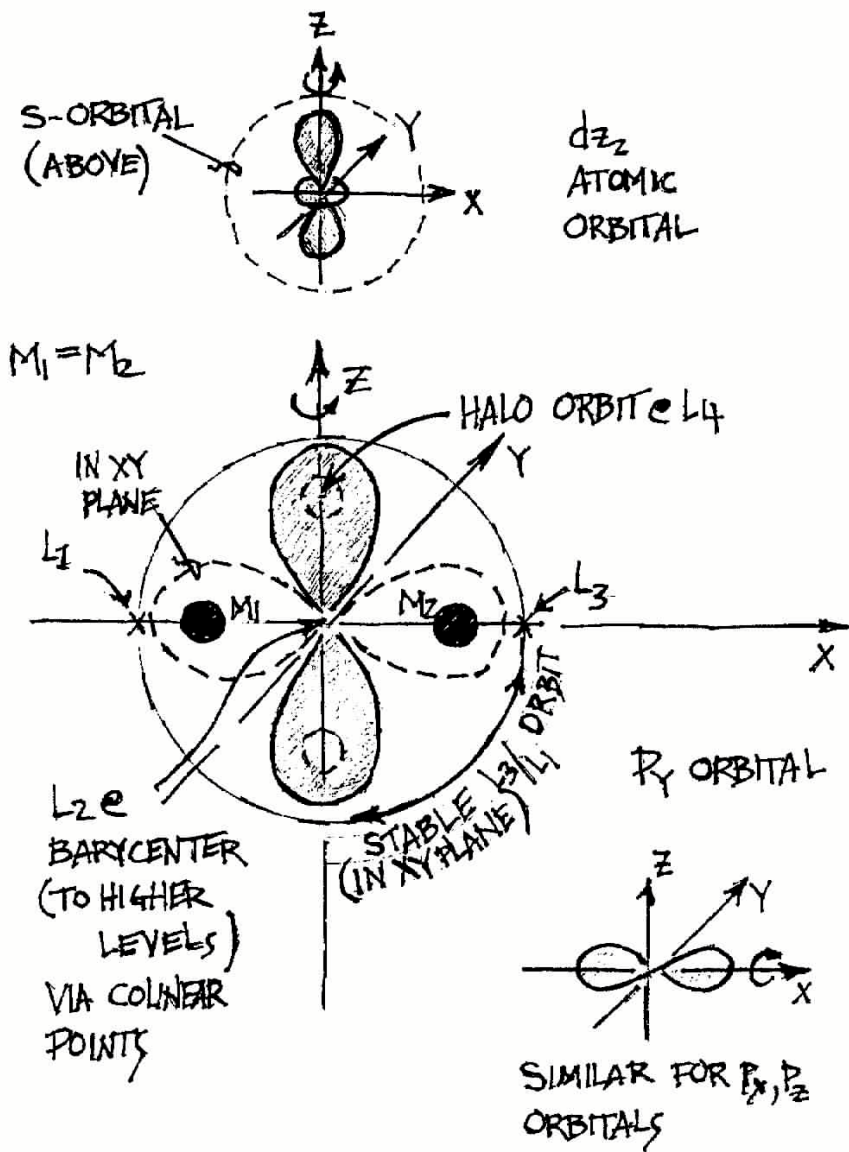


Free Return Orbit as an intermolecular Bond

This illustration shows two positively charged nuclei, with an electron orbiting between them along an equipotential surface. A body on such a surface, by definition, draws no work from the respective electric fields and thus is ideal for a bonding orbital.

The illustration shows the electron orbiting between the two protons, and beneath their respective atomic/electron shells. In this regard, the bonding orbital "shields" part of the nuclear charge (16), perhaps so that one proton in each nucleus is classified as a "neutron" because its electrostatic charge is expended on the bond.

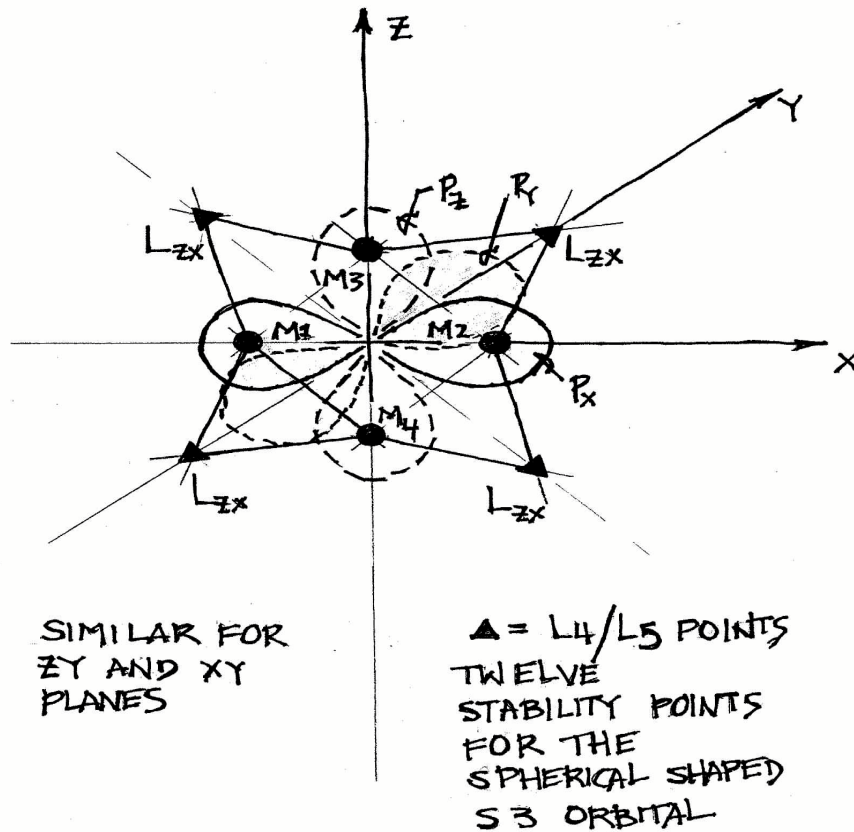
In the figure below the Three Body Problem between two atoms has the five Lagrange stability points (perhaps two more equilateral points in a system that is three dimensional), giving "anchor points" for the atomic electron shells (+/- electron spin?), with perhaps one L4/L5 pair for each unique atom.



Complex P Type Orbital Bonding

The general idea is to show how Lagrange Points make it possible to next orbitals, along the colinear axis which exists on both levels - the free return figure-8 loop at an L2 point actually forming an orbit between successive levels; the L1 and L3 points just serving as anchor points on either level.

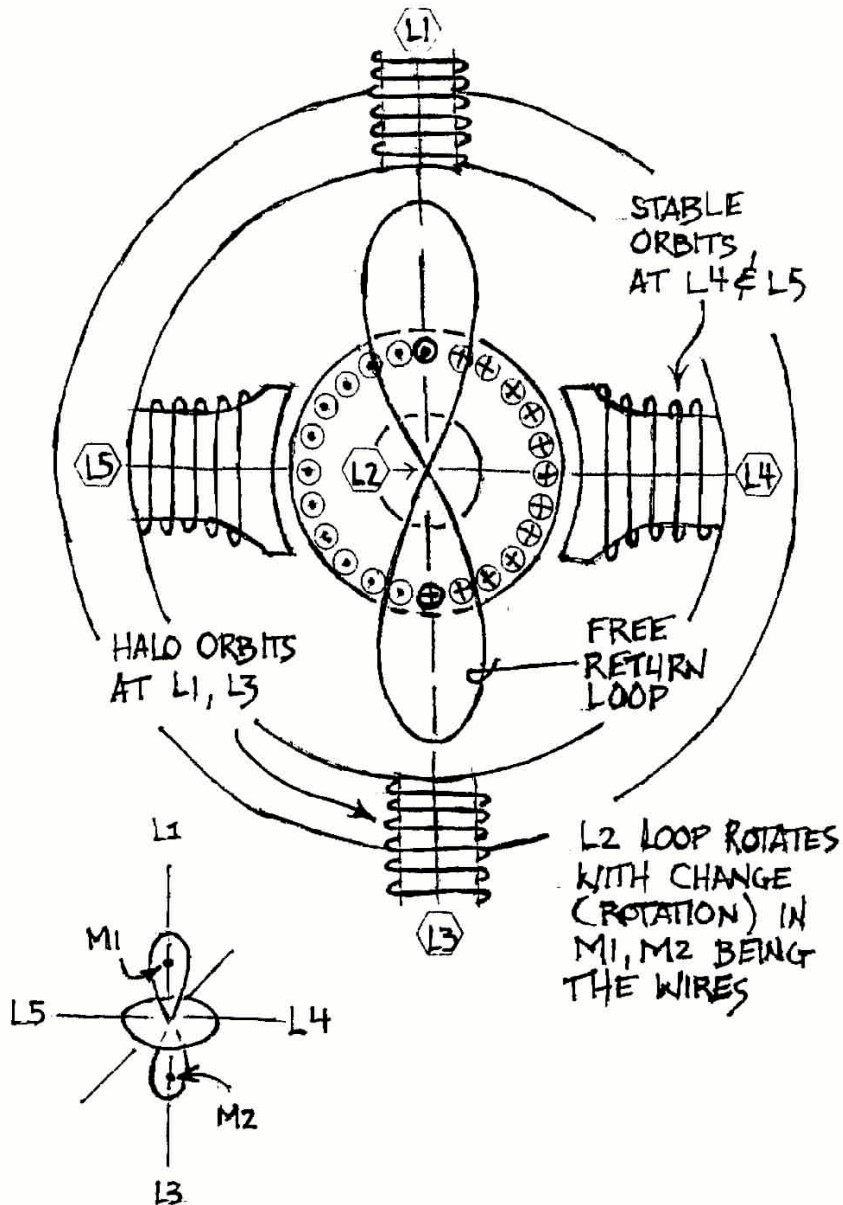
The relationship between successive levels forms higher level Three Body Problems, and create new Lagrange Points unique to the higher level.



Anchor Points at successive fractal levels

Consider that the quantum 3BP is such that L3 is beyond the barbell shaped orbitals. Then a halo orbit at L3 is a barbell orbit of the next level, i.e. the next higher energy level barbell shaped orbit.

The figure on the following page shows how all this might be modeled in the electromechanical motor/generator scheme that has been cited successfully before. The halo orbits are windings on the stator poles N-S-E-W, at the various Lagrange Points, and the L2 free return orbit centered at the polar axis of the rotor is the operating part of the model, existing at both levels.



Electromechanical interaction of subatomic bonds

V. Mathematical Constructs of Singularity

The general idea of mathematical physics is that the mathematics can be used to study physical processes. For example, the motion of a system of bodies can be represented by integrating the differential equations describing the forces acting upon the bodies. This research has gone one subtle step further, by suggesting that the mathematics is more than a model for some processes but the process itself; e.g. in the case of the Fourier Series describing the incremental steps of allowable energy in the general 3BP. (ONE)

Celestial Mechanics has an expression for this idea called "integrals of motion." They are laws that apply in restricted situations, in a small neighborhood of an optimal trajectory or solution. For example, consider the Jacobi Integral (total energy is constant) for the circular coplanar 3BP. It is a "parity check" matrix for non coplanar, non circular problems; just like the steps decided upon by the variable step integrator for the 3BP computer model. (TWO)

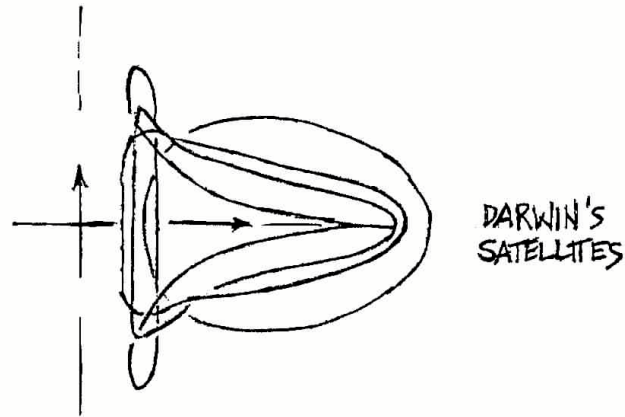
As an introduction to this purely mathematical section, it is interesting to consider the numbers that we know must exist in nature. The ratio of a circle versus its radius comes first by most accounts - the number 1 for radius, of a circle with circumference equal to pi. Strictly from the mathematical perspective, the fact that pi is an unknown and unknowable number (infinite number of digits, never repeating) implies that circles are not possible in nature - e.g. circular orbits - and perhaps they are mathematically equivalent to a point singularity, when posed in the right context (e.g. in the complex plane).

This, in turn, implies that nature acts to avoid circles - or at least to circumvent them whenever possible. A concept which leads inevitably to motion on the unit circle, which is appropriately represented by another unknown and unknowable number, "i" or $\sqrt{-1}$, the imaginary number. Where uniform motion on the unit circle - nature's way of representing a higher order singularity - is represented by $e^{i\omega t}$ - which makes "e" a third fundamental value of nature, also infinitely unrepeating.

Taking this logic one step further, and you surmise that a spherical orbit is also a singularity condition. That is, a thin spherical shell (or hoop in 2D) of uniform density reduces to a point mass - i.e. a singularity. Nature does its utmost to avoid motion in these type of systems, doing so by sinusoidal increments (sin/cos waves being the projection of a point rotating on a circle - if that circle moves, it forms a 3D helix). Theoretically, every motion of every body is a variation of the unit point/circle/sphere, in concentric domains of ever

increasing extent until ultimately only life itself maintains the dynamical stability of the fragile whole.

On the level of life, this stability can be studied, modeled, and understood using the mathematics. This section looks at a few important concepts in mathematics from the perspective of the dynamics of the 3BP - which is itself, in the final reckoning, a singularity of nature.



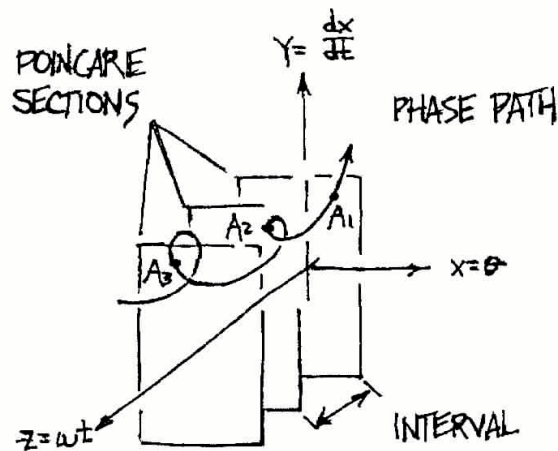
Singularity Constructs or 3BP Periodic Orbits

23 Poincare Sections

It is interesting to note that Poincare sections are a conformal mapping of concentric parallel slits in a circular domain. The key is to represent this as a bi-linear transformation, featuring an interplanetary trajectory. The problem is that the trajectory is going between two different fractal domains, which is illustrated quite nicely by this transformation because it is a combination of three separate linear transformations:

- translation (f-force at the sun)
- inversion (gravity)
- stretching (g force at Jupiter)

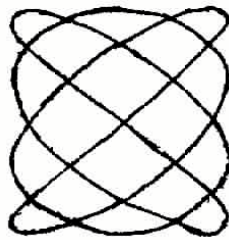
The following illustration of the "system wave" shows the motion of the planets to exist, each in a unique "Poncare Section."



Poincare Sections on Phase Space

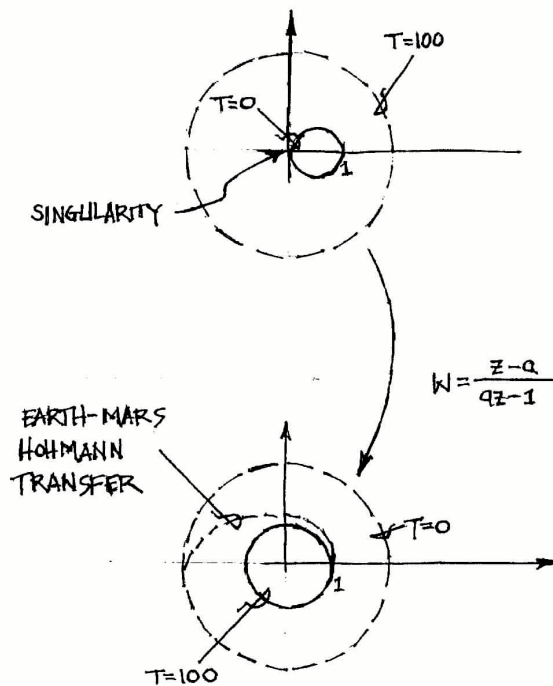
Closed noncircular periodic orbits can happen only with an inverse squared law force (i.e. gravity or electrostatic potential) or the harmonic oscillator potential which is a $1/r$ force \sim iff the ratio of the angular frequencies of the x- and y-motion is rational. In the mathematics these are called Lissajous orbits. (Note: the angular frequencies are equal for elliptical or straight line motion in the harmonic oscillator model.) "Commensurable" means the ratio is a rational fraction.

A typical Lissajous orbit, a closed periodic orbit, is shown in the next figure.



A Lissajous Orbit

Note that if the ratio of angular frequencies is different from a rational number by even an infinitesimal amount (i.e. both numbers must be known to infinite precision), then the motion is no longer closed and will fill up the square. Since pi and e are both irrational numbers, they cannot in any way be associated with a periodic orbit ~ thus confirming that circles are singularities. (FIVE)) Which is odd, considering that for all bounded functions of two variables, the value at the center is equal to its average value on the circumference of the circle ~ e.g. nature does its utmost to achieve circular motion, despite its instability.



The Biharmonic Conformal Mapping

The above illustration shows the instability of a one proton/one electron system (the Bohr Atom), with the electron in a perfectly spherical orbit - e.g. no perturbations. A spherical charge distribution, as with gravity, induces a zero charge field within the sphere - a point singularity. This destabilizes the proton, causing motion; having no counterbalancing electric negative shell, the proton nucleus field begins to expand; bumping the electron out to a higher orbital; and it stabilizes in a higher order spherical s-type orbit; and so forth. Conformally, the two conditions are the same.

24 Linear Algebra

The equations of motion for the Two Body Problem can be represented as the modal matrix

$$\vec{F} = [\vec{P} \quad \vec{Q} \quad \vec{R}]$$

then

$$\vec{F}(x, y) = \vec{P}(x, y)\hat{i} + \vec{Q}(x, y)\hat{j}$$

in two dimensions. Since \vec{F} is a conservative field, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

which actually does not work because \vec{F} is conservative only in the plane of the orbit. To see this create a hodograph with two separate circles for the bi-elliptic Earth to Mars trajectory. If \vec{F} were a conservative force then the intersection of the two circles would coincide, but it does not. To see why, consider the Cayley Hamilton theory, which says that a matrix satisfies its own characteristic equation. There is a contradiction that the f and g equations solve the Two Body Problem, but not the f and g matrix. That is, the fact that a matrix satisfies its own differential equation (the C H Theory) means the matrix is an envelope curve.

Note that the transformation to the symmetric plane is linear. This implies other systems can be approximated by a matrix (e.g. in orbital estimation theory) which is of a linear operation of independent vectors ~ that being the case, this system can be studied using linear programming techniques.

25 Optimal Controls

No matter what coordinate system you use or how the equations of motion are integrated, there is always a jump discontinuity at the sphere of influence (SOI). The only way to get a reasonable solution is to apply a change of coordinates there and to use statistical estimation methods to drive the system to approximate the solutions on either side of SOI - or to assign the discontinuity a penalty function and to drive the penalty function integral to zero.

$$J = t_f + \int_{t_0}^{t_f} a^2 dt$$

The problem is that it is not possible to drive this value to zero. Hence there is a new, unknown force at SOI. This implies the existence of a new integral of motion, which can be found by setting up the trajectory optimization through SOI as a convex or concave problem, which restricts the problem without loss of generality and eliminates a variable. This is an unlikely and unexpected result, brought about because the $\delta x = dx$ assumption in basic calculations of variations theory is OK for simple trajectories but not for interplanetary trajectories because they traverse a fractal change of level. This can be perceived by characterizing the extremal path of a low eccentricity ellipse as the variation from the unit circle.

26 Potential Theory

It is possible to represent gravity as a potential

$$\nabla G = U$$

This means that gravity is an irrotational field (conservative) and

$$\nabla \times U = 0$$

Which means that gravity is incapable of two of the phenomena commonly associated with it, the centrifugal and coriolis forces because they are both rotational forces.

On the other hand, electromagnetic theory states that an \vec{E} field is irrotational

$$\Delta \times \vec{E} = 0$$

if \vec{E} is due to static charge. While $\vec{F} = q\vec{E}$ and

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 R^2} = k \frac{q_1 q_2}{r^2}$$

which is exactly as the universal law of gravity. So, \vec{E} is analogous to \vec{U} but since it is well known that \vec{U} is not irrotational; then that means the \vec{E} associated with force from static charges per the inverse squared law for electrostatic force comes from charges that are not really static at all, at least on the quantum level. The corollary in gravitational terms is that the mass in gas giant planets is not static or uniformly distributed, but arranged perhaps in the kind of core and anomaly masses posed earlier.

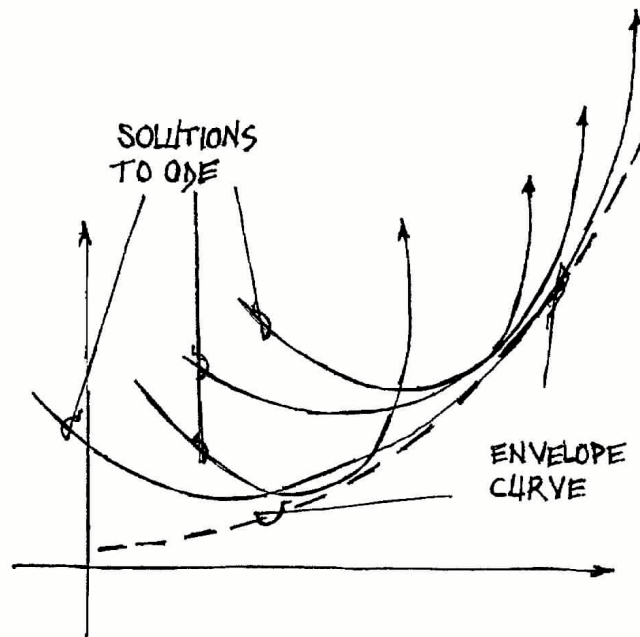
27 The Envelope Curve

The envelope of a set of functions is itself a solution to the problem. It is defined as follows:

$$\begin{aligned}\phi(x, y, c) &= 0 \\ \frac{d\phi}{dc}(x, y, c) &= 0\end{aligned}$$

Like the free return trajectory, the system wave, or the "wheels" that are its substructure; envelope solutions are not found by the actual equations of the system but by inspection of their solutions. It is assumed that these envelope curves represent a solution to the system from the point of view of the next higher fractal level and are consequently a kind of zipper linking two different coordinate systems together via a trajectory that is a solution in both domains.

Note that the same concept applies to a three dimensional surface, which is essentially what the "system wave" is. (3)



A System of ODE's with an Envelope Curve solution

A system of ODE's is said to have an envelope curve if (1) there is a unique member of the family tangent to it at each point of the envelope curve and (2) every member of the family is tangent to the envelope at a distinct point of the envelope.

In order for the "system wave" to be an envelope curve solution for the outer planets, each of them must occupy a separate loop. That being the case, the variation in intercept at the origin of the symmetric plane is what simulates this "loop" and so the planet is itself in a planar orbit. This rather implies that each of the outer planets must have a loop as part of its trajectory on the system wave.

It was stated in (21), and is implied in the scaling of the system wave, that the wave itself triplexes after Jupiter ~ thus three solutions coincident on the symmetric wave are OK to still be a family. This puts Jupiter on the same segment as the asteroids (which are on the inner planet segment), and results in a complete coverage of the envelope curve at that level.

28 Vibrations

Take the 2nd order Ordinary Differential Equation (ODE) for the three bodies and form a 3x3 matrix, set the determinant equal to zero, and solve for the eigenvalues and eigenvectors. The eigenvectors will be the stable inertial axes of the gravity ellipsoid (bonding orbital orientations at the atomic level). The eigenvalues will be the "buckling modes" or the rate of vibration at which these molecular bonds will break. A stable vibration about these bonding orbitals will result in the controlled emission of bonding energies by $E=mc^2$ - heat. There are probably some materials that will emit energy directly as EM waves, or that can be made to do so as an energy source.

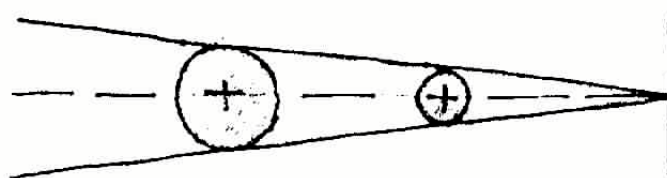
As a preliminary to the following derivation of the Law of Gravity used by Newton, consider the impact of Relativity - which adds a term to the Universal Law of Gravity

$$F = G \frac{m_1 m_2}{r^2} + R$$

What does this imply? Geometrically, Relativity says straight lines are not straight, or the shortest distance between two points; circles aren't circles, mass isn't mass, density cannot be uniform, and angles cannot be measured accurately. This obfuscates every principle of analytical geometry and every notion of common sense.

Gravity

It is really quite elementary, how Newton derived the Universal Law of Gravity (Moulton, p. 104). He used just one figure:



The two spherical shells attract a particle p (at the apex of the lines) equally because any solid angle which includes part of one shell also includes a similar part of the other shell. The masses of these included parts are as the square of their distances, and their attractions are inversely as the square of their distances. Thus, you get the above equation (without R).

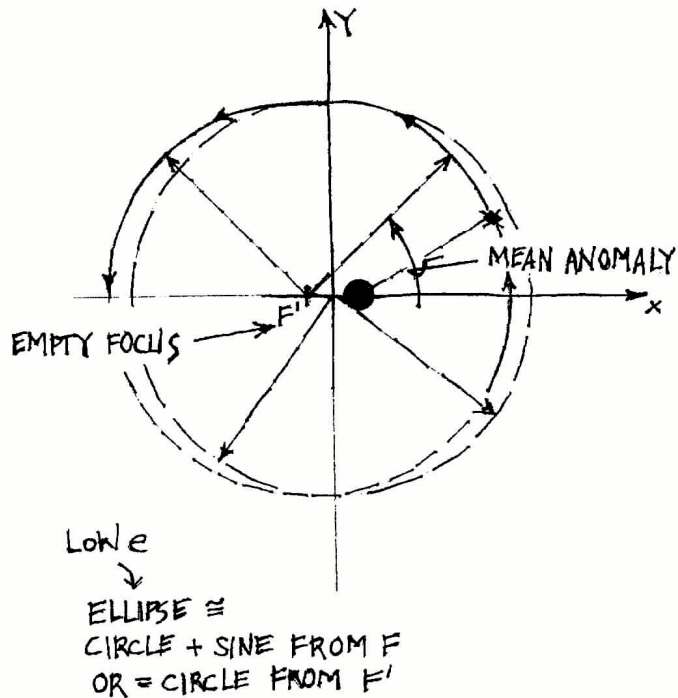
In the calculus, which Newton created later, you get

$$\ddot{r} + r = \frac{\mu}{r^2} \Rightarrow \ddot{r} = -\frac{\mu}{r^3} \vec{r}$$

The correction due to Relativity is

$$\ddot{r} + r = \frac{\mu}{r^2} + \alpha u^2; \alpha = 3 \frac{\mu}{c^2}$$

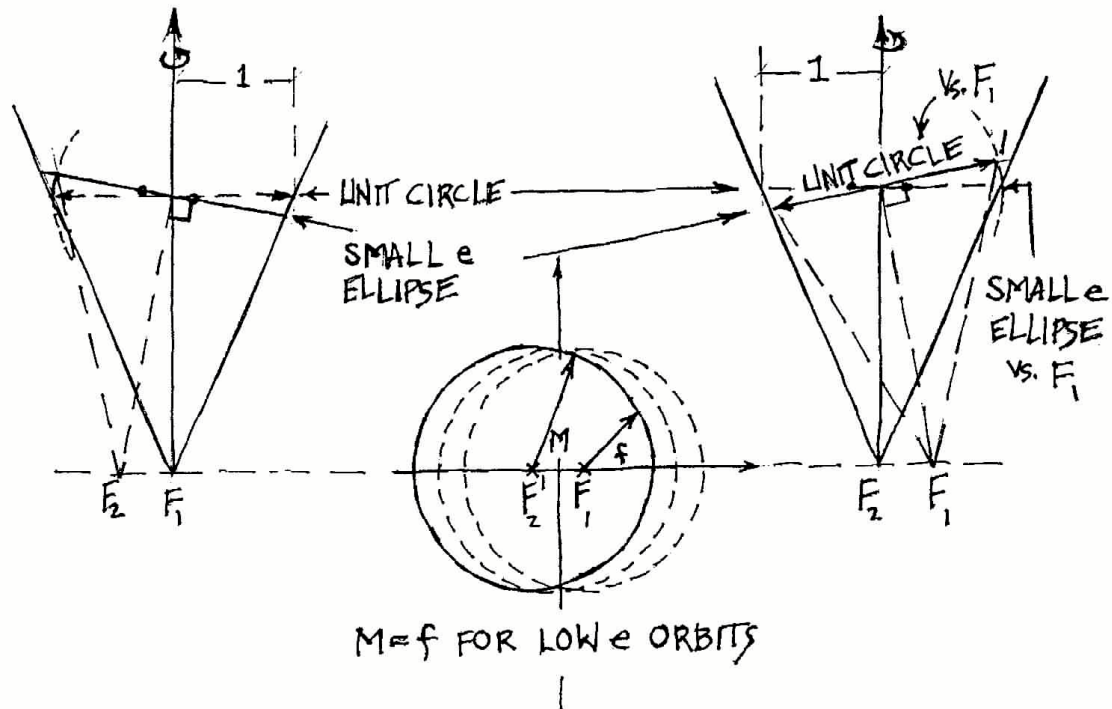
where c is the speed of light; so the correction is admittedly exceedingly small. However, this does alter the geometry of the problem in such a way that Kepler's 2nd Law works for eccentric orbits, even though the math is for circular orbits. Relativity, with this constant, presumes to change space AND time; when, in fact, the whole analysis here takes a much more rational approach, that time is invariant and only space is distorted because of extremely high gravity.



The Small Eccentricity Ellipse

The theory posed here argues for a geometric distortion in the neighborhood of the sun, for example, that for a small eccentricity ellipse puts the focus at the ellipse to be coincident with the geometric center of the ellipse, in such a way that the orbiting body is then

acted upon by the usual inverse squared gravity force, plus the much more subtle $1/r$ force whose action causes the precession of the perihelion of the inner planets, most notably Mercury.



The $1/r$ force at work

The above illustration shows this affect in great detail; whereas an ellipse is a conic section, and the foci are each so close to the geometric center of the ellipse; whence the peculiar motion of the origin of the symmetric plane makes the foci and the center coincident during much of the orbit, the remainder being negligible - zeroing out over the long term.

What this means is that Relativity serves to correct a simple error in geometry by causing the local distortion of space and time, and - moreover - propagates this unnecessary distortion through the space sciences by actual altering the most fundamental law of them all, the Universal Law of Geometry. This has had the affect of making all the basic sciences more and more abstract and statistical, less and less realistic and practical.

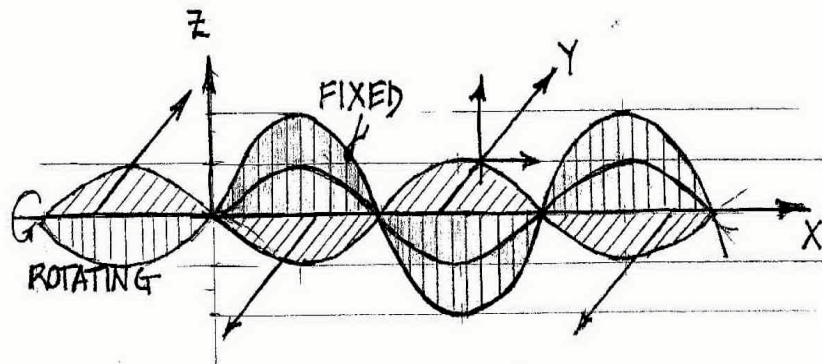
To whit, any violation of this law of Relativity must do so using equally simple geometry - e.g. Kepler's 2nd Law of Equal Areas, which has been shown to itself be flawed. It is not good science to justify a flaw with a flaw.

29 Seasonal Time Variation

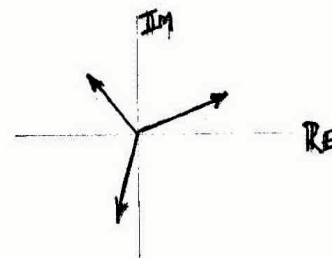
Two sine waves out of phase form an ellipse, at an angle to the principle axes they change the inclination of a circle and its plane of orientation. Add these two phenomena and you have an EM wave which transforms the unit circle into an elliptical orbit with all the usual parameters (6). This is how weak and strong gravity waves are oriented.

Motion caused by such a wave is symmetric with respect to the symmetric hyperplane. In the 3BP, the center of mass moves so that it remains on the z-axis - an up/down motion that causes the planet to rotate, like a spinning top pumped from the top.

Melting polar ice redistributes mass and alters the dynamics of the Earth. The reverse process - onset of an ice age (which we are past due, for one) balances the natural drift of the Earth away from the sun by temporarily (well, for a period of 80,000 years, the typical length of the last eight ice ages) increasing the strength of Earth's gravity waves, pulling it closer to the Sun. If global warming continues; well, Earth will not have its ice age, and it will keep right on drifting away from the sun.



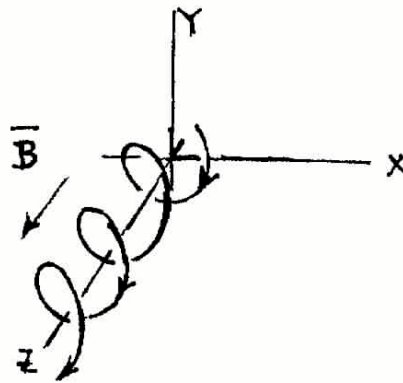
TIME DOMAIN



FREQUENCY DOMAIN

You will recall that Gravity exists as a point-to-point force only in the frequency domain. That is, between planets. Between solar systems or stars gravity exists as a shock wave, or as the boundary between frequency domains that in the time domain behaves like a gravity "string," hence the origin of string theory. These strings are nothing more than the static shock wave of intersection gravitational regimes; in which context gravity is nothing more than a geometric construct and not a force at all. It's the f- and g-forces that do all the real work.

Not to return to the electromagnetic field analogy to give some insight into this subtle phenomena. The motion of a charged particle in a uniformly constant **magnetic** field is in the shape of a helix (which, actually, is just the shape of the system wave):



Where \vec{B} is a static magnetic field. Some other properties of this field are:

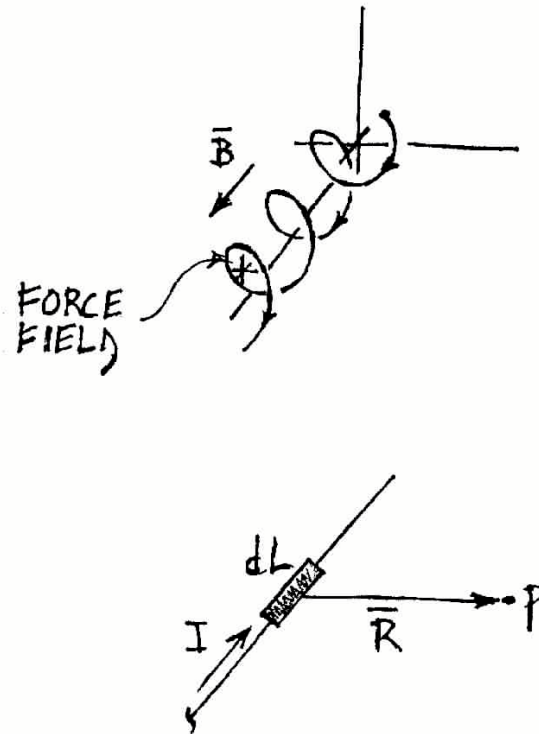
- ✓ Speed in the direction of the magnetic field is constant
- ✓ The angular frequency of motion in the circular path is independent of speed
- ✓ The project of motion in the xy plane is a circle, where

$$\vec{F} = q(\vec{v} \times \vec{B})$$

- ✓ A particle under the influence of this field moves at a constant speed
- ✓ Acceleration is always perpendicular to the direction of motion

Now to consider this scheme in the context of the theory of planetary orbits posed heretofore. If the system wave is in fact a helix in space, as was shown by numerical methods; and if this wave has some electromagnetic component, as was implied by how the axial inclination of the planets was linked to the system wave and to the geomagnetic aspect of the rotating planet; and if all of this is in fact but a figment of the Three Body Problem, then the earlier

conclusion that all these forces are coordinated nicely by an L6 Lagrange Point existing in the middle of a loop; then, the above figure can be modified slightly.



And the mathematics show the field is as if generated by a small segment of charge on a straight line, upon a distant point ~ the linearization of the action-at-a-distance unit ball that enacts the $1/r$ force locally. Consider the following :

- ❖ This is all the same as Coulomb's Law, except for the direction of the field
- ❖ It's exactly what happens with a halo orbit around a Lagrange Point, in which the body acts as though it is orbiting an empty focus
- ❖ Thus, the "weak" gravitational force alleged by the U.F.T.

And finally the motion along the line is the dynamical element of the whole problem that has evaded this analysis so far, some consistent change in the field versus the planet at point p , that happens as the solar system evolves.

30 The Uncertainty Principle

A fundamental flaw to quantum chemistry and the Schrodinger Cloud Plots is that there is no provision for how an electron goes from one stable orbit to the next. Going from a circular s-type orbit to a barbell p-type orbit is a very complex maneuver involving plane changes, Hohmann Transfers, and Lambert Transfers - any one of which requires far more energy than the difference between the two orbital levels. Basic orbital mechanics shows the whole complex scheme embraced by atomic physics is unworkable unless there is a single fundamental frequency that exists like a common harmonic between all allowable energy levels - e.g. the "system wave."

Now to consider some important mathematical ideas:

- Each physically observable property in quantum mechanics (e.g. position, velocity, energy, momentum,...) can be represented by a Hermitian operator, typically a matrix
- Two operators with the same eigenvectors commute
- Physical variables with noncommuting operators cannot be measured simultaneously with arbitrary accuracy
- Position and momentum operators in quantum mechanics do not commute; this is the Heisenberg Uncertainty Principle, which says

$$\Delta x \Delta p \geq h / 2\pi \Rightarrow h = 6.62377E - 34 \text{ joule-seconds}$$
- The existence of a matrix transformation for the symmetric plane, where eigenvectors do commute, means that the Heisenberg Uncertainty Principle does not apply in dynamical systems whose coordinates are transformed into the symmetric plane.

Hermitian matrices have the following properties: (i.e. $A^{-T} = A^H$)

- Diagonalizable, i.e. $Q^{-1}AQ = Q^T AQ$
- The eigenvectors are real
- The eigenvalues for unique eigenvectors are orthogonal to each other
- A matrix with complex elements plays the role of a symmetric matrix - i.e. they are equal to their conjugate transpose; $a_{ij} = \bar{a}_{ji}$
- The diagonal entries are real (implying the kind of symmetry that was seen with the symmetric plane)

The principle axis theorem of mechanics applies to orthogonally diagonalizable matrices:

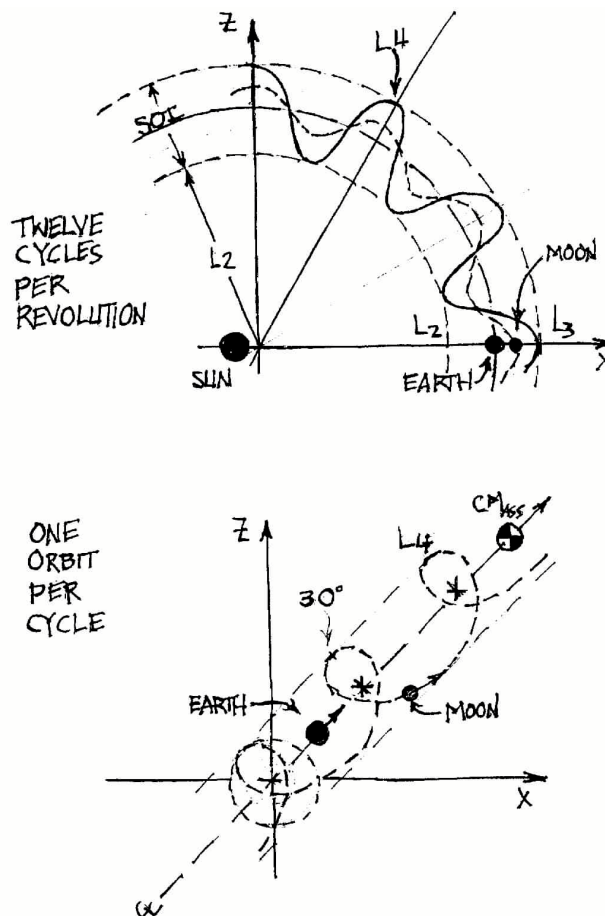
- If the eigenvectors define the axes, then the axes are perpendicular
- In deformable body mechanics, the eigenvectors give the directions in which there is pure compression or pure tension. In other directions, there is pure shear.

This is just the phenomena that was implied by the Mohr Circle analysis earlier.

31 Quantum Chemistry

Chemistry is fundamentally statistics - e.g. the electron cloud plots are nothing more than probability distributions and tell you nothing more than a cloud tells you about any single particle of water vapor inside of it.

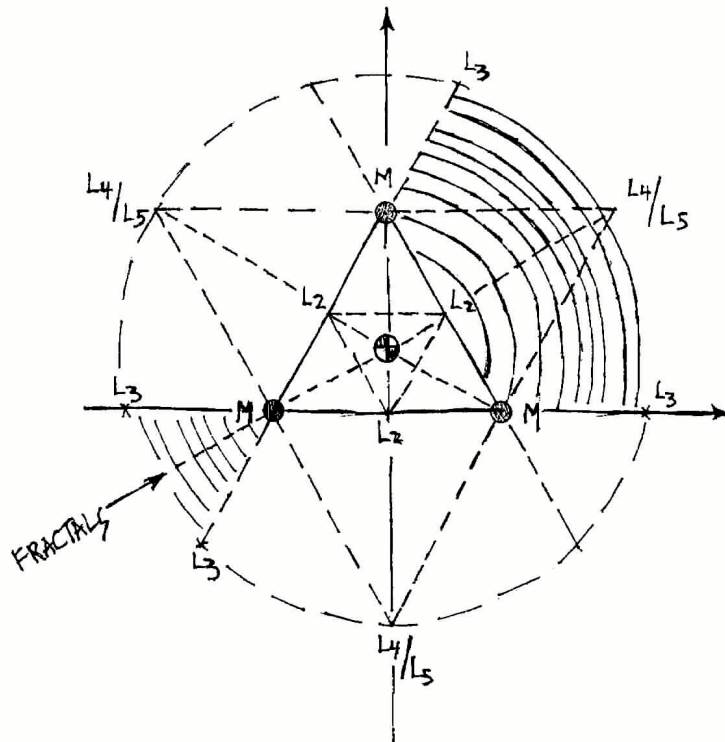
The problem is that angular momentum is not represented mathematically correct in analytical chemistry. They use the "angular momentum vector" to talk about electron spin and other phenomena. However, angular momentum is not just a vector when considered in the context of orbital motion, but also a plane. You cannot have one without the other.



Moon's orbit as a halo orbit around a L6 Lagrange Point

This is a dilemma because there is no clear orientation in the Schrodinger distributions, as is expected for any determination based upon the angular momentum vector. In fact, there should be a whole series of orientations for the nested cloud plot patterns, but all are absent or nowhere accounted for. This is a dramatic flow in theory, but perhaps the fluxion hierarchy can resolve the problem.

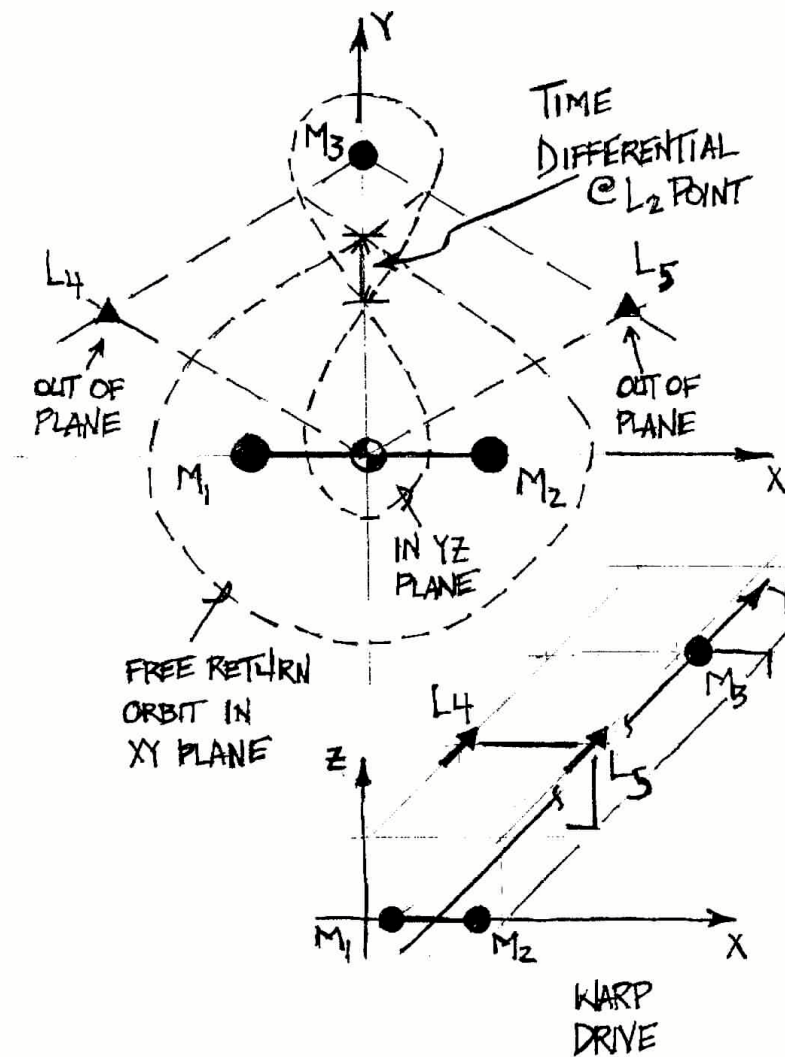
Consider the three phase AC current generated in a rotor with three masses oriented as on an equilateral triangle.



The stable points for these three masses are all located on an exterior circle, as shown. Increase the separation of M3 and the circle is broken, but the continuum of Lagrange points is retained at first and the circle warps into an ellipse, then a Lissajous curve, and finally into a helix. There are twelve "critical points" or "anchors" just like the Earth moon (twelve cylinder rotary aircraft engine) depicted previously.

Operating the twelve phase motor between charged plates allows directional control of the helical waves, as indicated. (14) The rotating three phase rotor creates an EM wave.

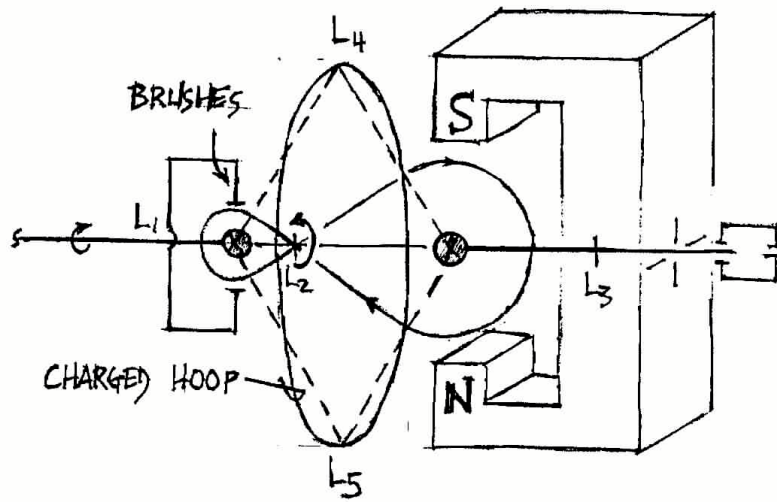
The symmetry of the L2 Lagrange points induces a system attractor in the center - gravity - which is the axis of propagation of the wave in free space. A non dissipating laser. Reverse the polarity of the plates and you have a tractor beam. Organize it all around a three dimensional superstructure, and you have warp drive.



Conceptual Design of a Star Trek Starship

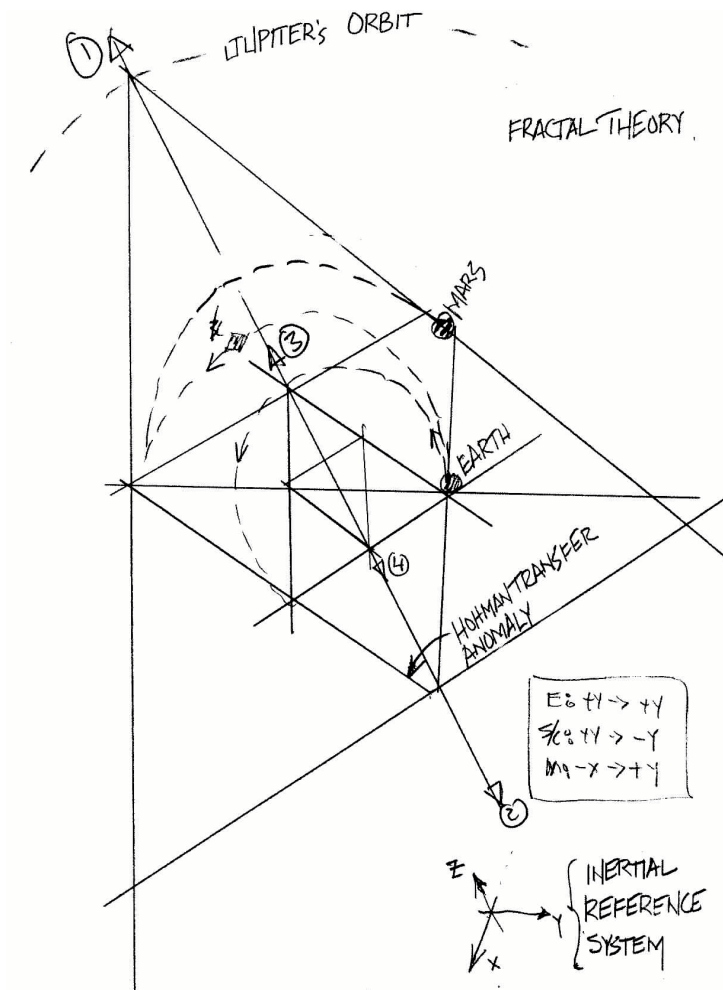
One final note about the peculiarities of the moon's orbit. There still seems to be no clear reason why the moon rotates 1x/orbit so that the same face is toward the Earth at all times. Perhaps this is because it's new to the solar system, and is still in "capture" mode or

it's so situated because it was formed quite recently from out of the cosmic cloud, or from the cloud of a disintegrating planet in the vicinity whose larger particles are the asteroid belts. Or maybe the action of the L6 Lagrange Point causes this, as with Mercury (3); almost.



32 Fractal Theory

There are a few unresolved issues: (1) how do the fractal levels intersect, (2) how do you go from the 3D Schrodinger Electron Cloud Plots to the 2D "galactic atom" and (3) what's the orientation of the coordinate system that organizes the fluxion accumulation that allegedly makes up the Fourier Series structure of Earth's gravity field. Amazingly, all three of these important questions are answered by a single illustration.



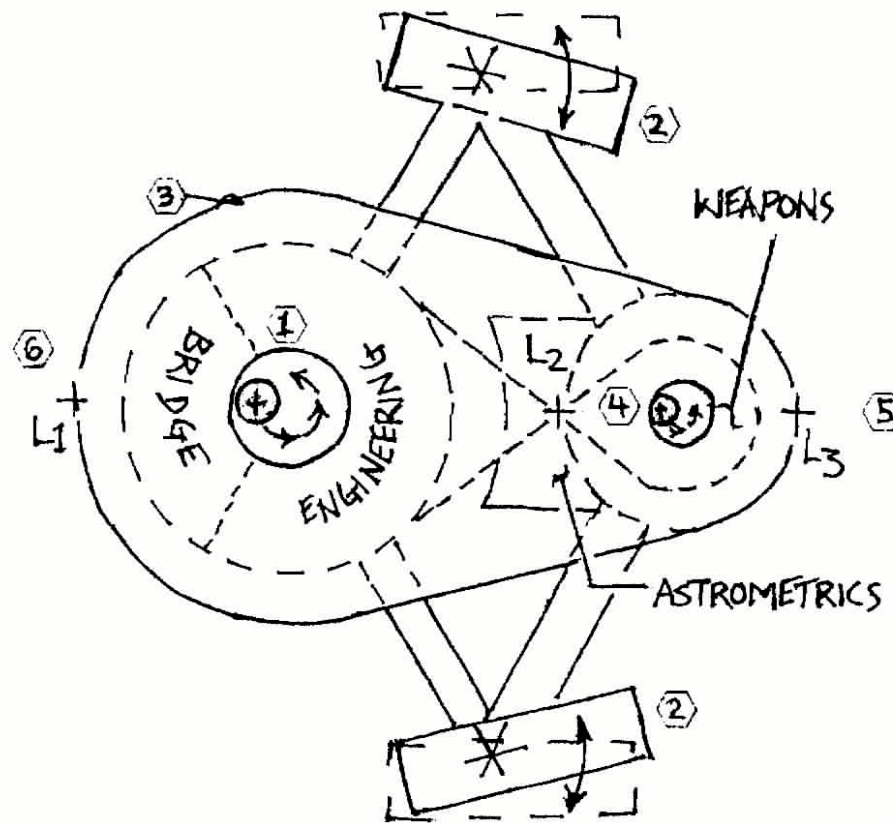
- (1) The figure shows the intersection between the inner and outer planets - a fractal division according to the system wave. The intersection is stitched together by nested equilateral triangles, symptomatic of the 3BP.
- (2) The coordinate systems between fractal levels are orthogonal, and oriented perpendicular to the axes of the stable equilibrium orbit triangles. In the case of the solar system, Earth's orbital plane crosses the symmetric plane on about Halloween (New Years Eve for pagans) and it crosses the invariant plane about three months later; the two lines of nodes are orthogonal, being separated by ninety degrees of heliocentric longitude.
- (3) The 2D/3D transition is also by this fluxion coordinate system, shown in the 3D transfer from Earth to Mars by a spacecraft - not much of a projection above the plane of the page, but perhaps the first increment in the inclination direction for staged transitions.

Also consider the dramatic bending of the arcs for Earth, Mars, and the spacecraft. The illustration is scaled (via Fractal Theory) down to fit the sun at say Earth's orbit for the outer corona. This suggests the bending of light may be different for different planes of approach to the sun (e.g. the sun's gravity does not emanate uniformly in all directions, but only in a fraction of the whole sphere - the plane of the solar system - a very energy efficient mechanism). Applying fly by geometry to this situation, it suggests anomalies internal to the sun. Alternatively, this may be related to the wave/particle duality of light.

Notice how the colinear axes intersect at right angles. Perhaps halo orbits at a colinear points are inter level hinges, flexible bonds between successive dimensions (unstable in either dimension, but stable between them) and halo orbits around L1 and L3 are inter dimensional free return orbits. Perhaps L4/L5 points are "particles" and L1, L2, and L3 are "waves" in the non rotating system.

VI Conceptual Design of a Starship

This is not a fantasy engineering dream, presenting make believe ideas, but a serious look at the technical challenges of building a space craft to travel beyond our Solar System.



Integrated Engineering Systems

The items in the figure above are familiar to Star Trek fans:

- [1] Warp Drive
- [2] Nacelle; aligns to gravity helix
- [3] Navigation Array

- [4] Aft Stabilizer
- [5] Weapons Axis
- [6] Weapon Directional Control

The space ship design is superimposed upon the geometric structure of a Three Body Problem, and the items refer to important concepts in Celestial Mechanics:

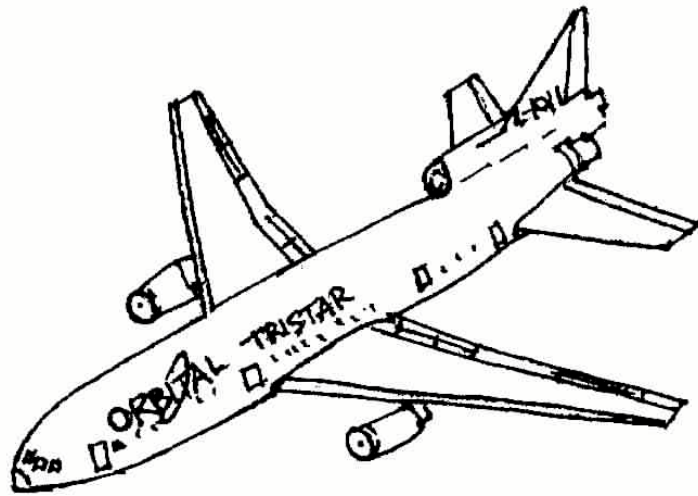
- [1] Barycenter
- [2] The L4 and L5 Lagrange Points or the Equilateral Points
- [3] A "Free Return" orbit
- [4] Small primary mass
- [5] The L1, L2, and L3 Lagrange Points or the Colinear Points
- [6] Halo orbits

This section introduces you to some new ideas and how they are integrated into the spacecraft design. The theory comes from Celestial Mechanics and the famous Three Body Problem (3BP). The "3BP Theory" tab on the side bar is where you should start. It describes this fascinating body of work, its applications, and its role in the design. It is not hard to understand this theory, and requires no rigorous math or other technical knowledge.

There are references to individual papers in the formal Theory which is in Tabs one through five, located on the horizontal bar menu at the top of the page. These are a series of forty short technical papers that generally get more mathematical as the number increases. Every paper, however, is carefully illustrated and can be grasped conceptually without any advanced mathematical background.

The starship design goes hand in hand with a comprehensive new model of space. The space craft, for example, does not have a fantastic new power source capable of generating huge amounts of thrust to achieve ultra high velocities. Instead, it is designed to fit exactly into the very fabric of space, like a zipper moving along an intricate series of interlocking links. The space ship finesses its way to high velocity, following the geometry of space like a surfer catching a giant wave, or a sail boat in a strong head wind.

The L-1011 Tristar section shows how the systems can be incorporated into a regular old passenger jet, the Lockheed Tristar. This is the platform where the ideas here can be developed and refined. The Light Barrier section shows that space behaves just like a fluid at very high velocities, so it's even possible that the same control surfaces that allow jets to fly on Earth will have similar purposes in deep space.

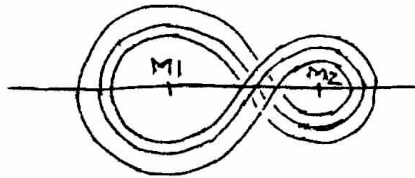
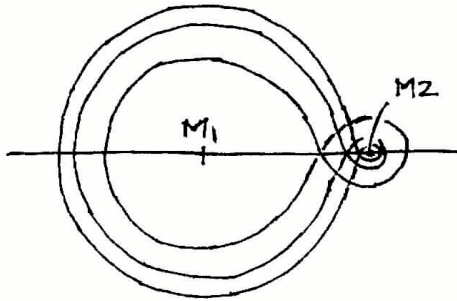


Lockheed L-1011 Tristar Jet

The formal Theory shows that gravity is a helical wave which collapses into a series of interlocking helix's - like three phase electrical current in a round wire. The starship nacelles create an electromagnetic field that locks the spaceship into this helix, like a train on a roller coaster ride. The Theory shows how this gravity helix acts upon the planets in our Solar System, and this gravity structure is extrapolated to deep space beyond the outermost planets. At this point the Theory acts just like the well known String Theory, which says that gravity is not a field across the vast distances of deep space but behaves more like an individual web made up of stranded strings.

The astronomical theory on the structure of the Solar System explains in a single consistent Theory dozens of phenomena in the motion of planets and their moons that is not explained by any other theory - including the orbital elements of all the planets, the rings of Saturn, Jupiter's "Great Red Spot" and why Earth's moon always shows the same face to Earth. All this is based on the 3BP of Celestial Mechanics, and is developed carefully, deliberately and with many illustrations.

Now, the equations of motion of a body under the influence of gravity have exactly the same form as the equations of subatomic motion between charged particles like electrons and protons. Consequently there is a 3BP of subatomic physics and also of quantum chemistry. The gravity forces that organize the planets in our Solar System and keep planets in stable orbits are just like the electrostatic forces that keep electrons in orbit around nuclei and form bonds between individual molecules.



Free Return Loops for Different Mass Ratios

What this means is that this elaborate new Theory not only gives new insights into the cosmos, but also into the very structure of matter itself. You may be familiar with all the atomic orbital shapes from high school chemistry. All these patterns are duplicated in special cases of the 3BP, and this suggests ways that matter can be converted into energy as a power source or for other "science fictional" applications. The above figure shows several types of stable orbits around two bodies by a small third body (hence, the term "Three Body Problem") that in three dimensions are barbell shaped - just like atomic orbitals.

Finally, the Theory here makes liberal use of the well known properties of Fractal Theory. This is the idea that patterns repeat, at ever increasing orders of magnitude. For example, if there is a 3BP at the astronomical level and one at the subatomic level, then so too must there be similar schemes in between - and beyond.

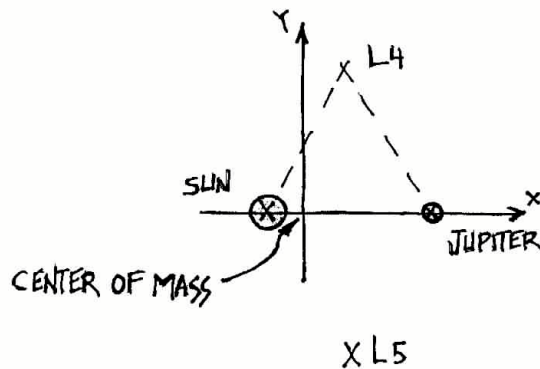
All of this probably seems quite abstract and so the next section shows what the classical 3BP is all about, gives some examples and then illustrates these ideas in some

practical applications. This process takes place throughout the project - theory, examples in nature, and then engineering applications.

33 Three Body Theory

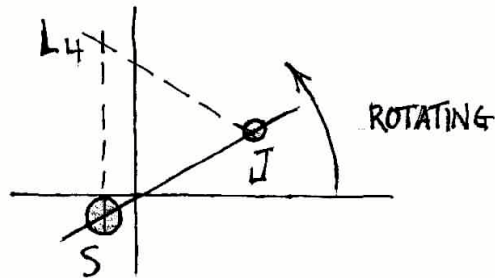
The most famous unsolved problem in mathematics is the Three Body Problem. All the famous mathematicians have tried to solve it - from Euler and Laplace to Lagrange, Jacobi, Poincare, and Lemaitre. Solutions are known to special cases - e.g. two large bodies in circular orbits and a third small body, all in the same plane - but the general problem of three bodies in random motion has no known solution.

This website considers one special case, called the circular coplanar problem. Consider one example, the Sun-Jupiter-Asteroids system (assume Jupiter is in a circular orbit, which is not too far off because its orbit is of very small eccentricity).



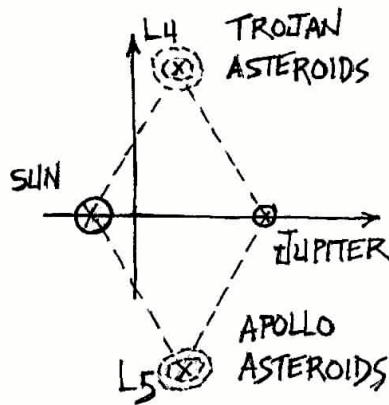
The Three Body Problem

If you draw a line from the sun to Jupiter, the center of mass or balancing point between them is near the sun. This is called the barycenter. Now make a coordinate system at this center, and let it rotate so the x-axis follows Jupiter around the sun. This makes the sun orbit the barycenter in a small circle and Jupiter in a very large circle. The sun and Jupiter are the two primary bodies. The primaries are always very large masses and the third body is much, much smaller by comparison - e.g. a satellite orbiting Earth in the Earth-moon system.

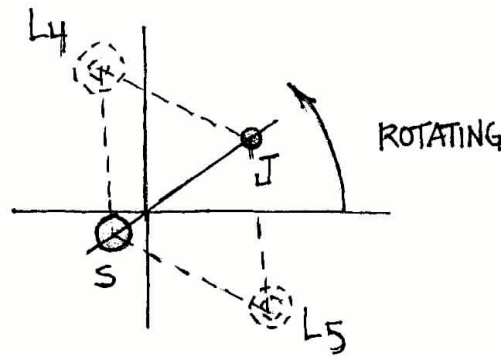


The 3BP is in Rotating Coordinates

There are five important points in the Three Body Problem (3BP) called Lagrange Points, numbered L1 through L5. The L4 and L5 points are stable equilibrium points and are situated at the apex of an equilateral triangle with one side formed by the primary masses. A small body at L4 or L5 will stay there, indefinitely. The small mass can also be in a small orbit around the L4 or L5 point, and will remain in that orbit even if there are small forces acting upon it. In the sun-Jupiter system there are small clusters of asteroids orbiting L4 and L5 and they are called the Trojan and Apollo asteroid groups.

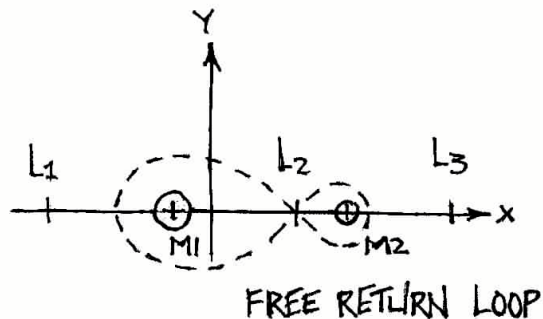


The Sun-Jupiter-Asteroids 3BP



The Trojan and Apollo Asteroids Rotate with Jupiter

Keep in mind always with the 3BP that the coordinate system is rotating along with the primary bodies, so that the two big masses always remain on the x-axis. That means the Trojan and Apollo asteroids always keep the same triangular orientation to the sun and Jupiter, and in fact have the same period of revolution around the sun as Jupiter.

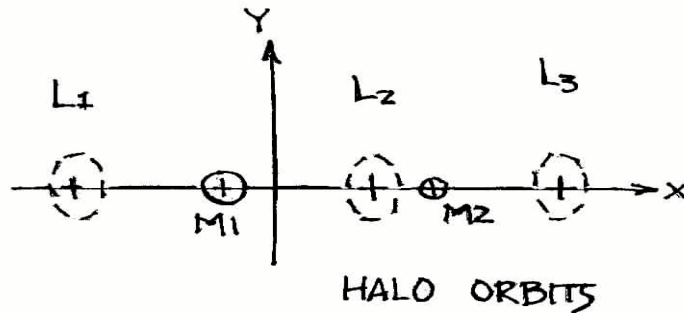


The Colinear Lagrange Points

There are three more Lagrange Points: L1, L2, and L3. The figure above shows an approximate location for the Earth-moon system. There is always a figure-8 shaped closed orbit (also called a periodic orbit) around both primary masses and through the L2 point. This is called a "free return" orbit and early Apollo missions to the moon followed exactly this trajectory because if anything went wrong (as it did on Apollo 13; on the far side of the moon,

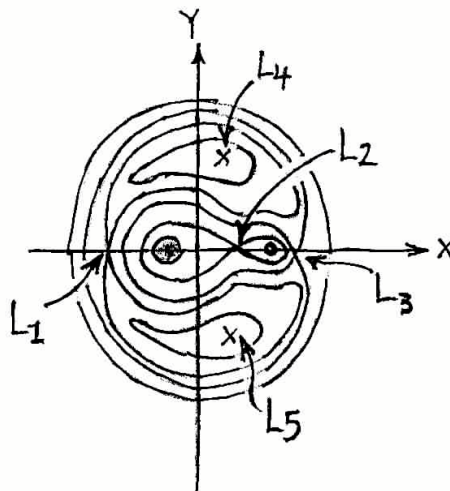
where their engines lost all power) then the spacecraft will stay right on the loop and return directly to Earth without the need for any thrusts or expenditure of fuel.

These three Lagrange points are unstable equilibrium points. An object can orbit one of them but if there is the slightest perturbation, or disturbance, to the orbit then the small object will drift out of the orbit and never return. Satellites are in these "halo orbits" at L2 and L3 to study the moon from a fixed altitude - and occasionally in an L1 orbit as well.



Stable Orbits around the Colinear Lagrange Points

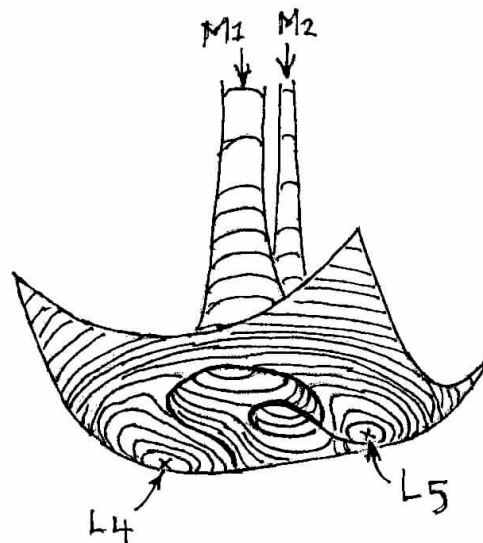
These five Lagrange points (named after the famous French mathematician who first solved the problem) exist for every 3BP, even if the two primary bodies are stars. The L1, L2, and L3 points are sometimes called the colinear Lagrange points and the L4 and L5 the equilateral Lagrange points.



Zero Velocity Curves

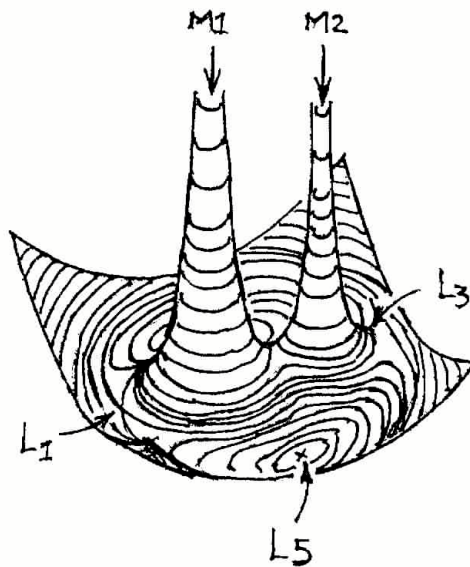
It may help to understand these points a little better if you look at a plot of what are called the zero velocity curves.

You can see the two primary masses, and all the Lagrange Points. Notice also the free return loop that intersects at L2. There are several other periodic, or closed orbits, if you look carefully. These curves are just like contour lines for a hilly area on a land map. A spacecraft with a specific velocity, or amount of total energy, cannot cross a line - just like a person hiking the hills follows a line of constant height. The L4 and L5 points are in small valleys - if you move away from either of them it is uphill in every direction, so if a ball is located there it will always roll right back again. That's why they are called stable equilibrium points. The other three Lagrange points are much more precarious because those points are at saddles - the slope is both up and down around them, so if you move away in any direction, you keep right on going and do not return.



3D Mapping of the 3BP

The above illustration is a three dimensional plot of the gravity around two large masses. The previous figure on the zero velocity curves is just a cross section of this three dimensional image in the plane of the two masses.



The Subatomic 3BP

There is also a Three Body Problem on the atomic level because the equation for the force of gravity and the equation for the force between charged particles like electrons is exactly the same except for the constant. The original atomic model, the Bohr atom - one electron orbiting on positron in the nucleus - came from Celestial Mechanics, and is what we call the Two Body Problem.

Not much will be said about the subatomic 3BP here but it is explored at length in the Theory. There the free return loop becomes a molecular bonding orbital, and Lagrange points are bonding points between and within molecules. Everything that we know about the 3BP in astronomy has an equivalent in subatomic physics. This is important because it suggests profound new ways of looking at matter - perspectives that could conceivably lead to such science fiction devices as the medical tricorder, the material transporter, or food synthesizers.

About the 3BP

That's all the theory you need to follow this section, and in fact not much more is needed to follow the Theory because everything is based on the 3BP. Just remember that the three bodies are in the same plane, two large bodies and one very small third body; the

coordinate system is rotating at a constant rate, and there are periodic orbits that follow a closed, repeating path.

34 Faster Than Light Theory

The biggest problem with Star Trek spaceships, and many other science fiction epics, is that Relativity says travel at - or even close - to the speed of light (denoted "c") is not possible. However, Relativity is not universally accepted by all scientists. There are many phenomena that do not fit Relativity Theory and it is coming under increasing pressure.

Relativity was not the accepted scientific basis that it is now, but was very much in contention well into the 1960's. There were equally plausible explanations in Celestial Mechanics, for several phenomena that "proved" Relativity theory was right:

The precession of Mercury's orbit

The bending of light by the sun's intense gravity field

A miniscule time divergence in atomic clocks orbiting Earth

Relativity is the standard today because a committee of scientists decided in its favor, over Celestial Mechanics. There remains a small but obstinate cadre of scientists who oppose Relativity even to this day. Mainstream scientists consider us "The Dark Side of the Force."

Relativity Theory itself was derived forty years before Einstein, by the prominent Celestial Mechanics worker, Henri Poincare. The only difference is the Poincare's equations assumed the existence of an aether (a super thin "vapor" in deep space), whereas Einstein derived the exact same result, using electromagnetic concepts. Most people in the modern day consider the notion of aether to be old fashioned and archaic, like the Earth being flat or all heavenly bodies orbiting Earth and not the sun. The truth is much more subtle.

There is a famous problem in classical Celestial Mechanics called "The Many Body Problem" (or the "N Body Problem," N signifying many or infinite in mathematical notation). The equations of motion of a small body acted upon by many bodies - e.g. an object moving rapidly through a solar system or several star systems with many bodies - reduce right down to the equations of incompressible flow in fluid dynamics. That is, the small object behaves exactly like it was moving through a fluid like air or water. This is an interesting result because it implies that at slower velocities, this incompressible fluid behaves like a very thin fluid - just like an aether. There is no similar analytical proof in Relativity, consequently the version derived by Poincare is much stronger mathematically.

Relativity says that matter cannot reach the speed of light because mass becomes infinitely heavy. Yet, every day electrons go that fast - if not faster, in electrical wires, in computers, and in countless electrical devices. No Relativistic effects have ever been noted in electric current flow or computer/transistor theory. (21) ~ numbers in parenthesis refer to other papers on the site.

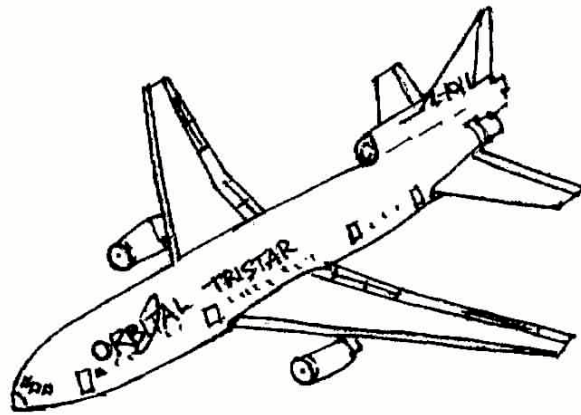
Many other facts are posed here, and a couple of brand new theories, that further erode the pre eminence of Relativity, and the likelihood that Relativity is just a special case - and not an all encompassing law. The implication is that the speed of light is not really a limiting constant, but simply a transition - from a thin vaporous aether, to a compressible fluid like air or water.

Over all there is at least enough solid mathematical evidence to say that travel beyond the speed of light is possible; and especially that travel at a high percentage of the speed of light is possible. This theme is recurrent through this project, which offers simple geometric and trigonometric explanations for the things explained by Relativity in extremely abstract terms.

Perhaps the strongest evidence in favor of this mechanistic perspective is that it explains many phenomena in our local stand still human level Solar System that are many orders of magnitude beyond the tiny forces supposed by Relativity. Moreover, this mechanistic Theory, when extrapolated, shows Relativity to be a special case; hence, more of a footnote than a full fledged phenomena.

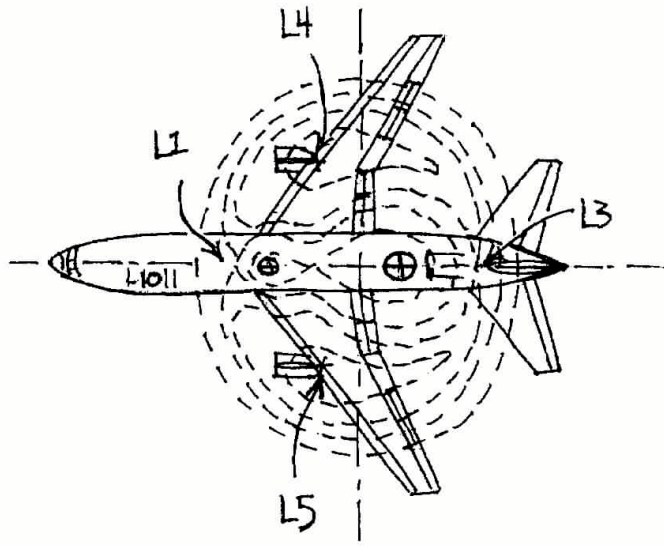
35 L-1011 Tristar Platform

The Lockheed Tristar has been in commercial use for over thirty years. There are still close to a hundred of them in regular operation - as passenger jets, airborne research labs, cargo jets, and even as mobile hospitals. Orbital Scientific has even configured one to launch a Pegasus missile into space, to insert satellites into low Earth orbit.



Orbital Scientific's specially outfitted Tristar

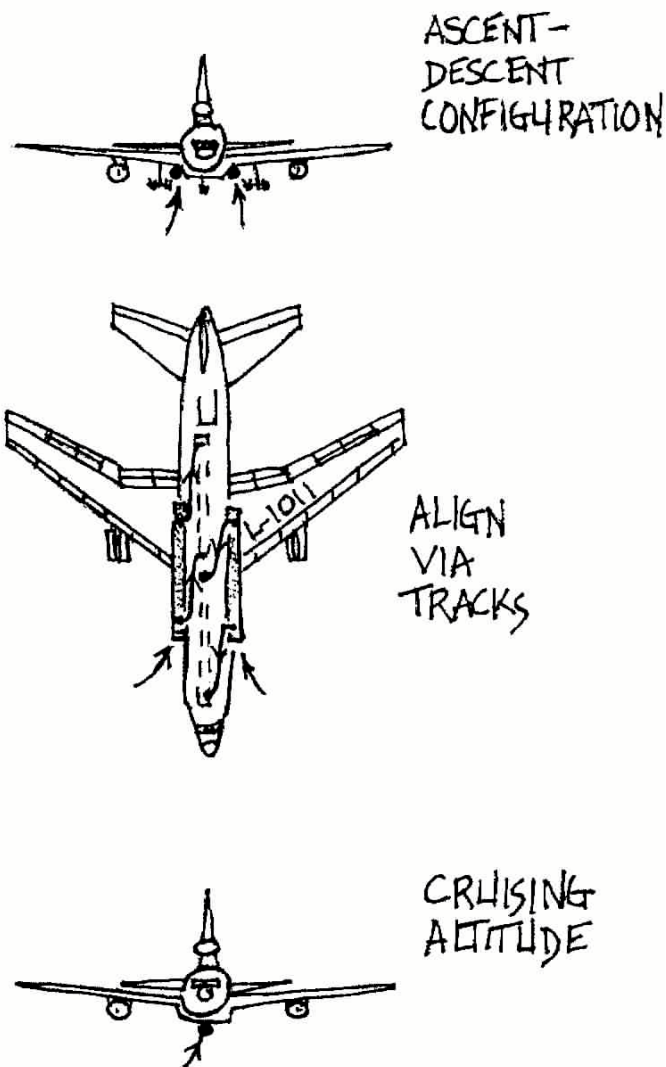
The Tristar has three Rolls-Royce jet engines, one on each wing and a third in the fuselage in the tail assembly. The air intake for this third engine is ahead of the tail on top of the main body of the aircraft, ducting air to the engine itself which is in the tail assembly aligned along the centerline of the aircraft.



Fuselage/Wing Structural Integrity

Applying the Three Body Problem ideas developed so far, you can overlay the zero velocity curves, with L4 and L5 points at the engine thrust points and the two primary masses along the centerline (approximately where cargo bays are located in commercial passenger jets). You will recall that in the 3BP, stable orbits are possible around L4 and L5, and quasi stable orbits around L3. This means that if all the mass of the aircraft is carefully balanced, then at high altitudes the gravity configuration will make small orbits at these Lagrange points very energy efficient - i.e. the turbine blades of the jet engines will operate very efficiently, and for all three main engines.

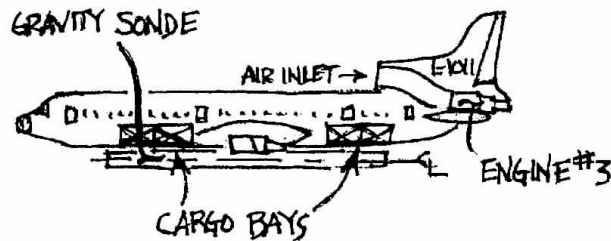
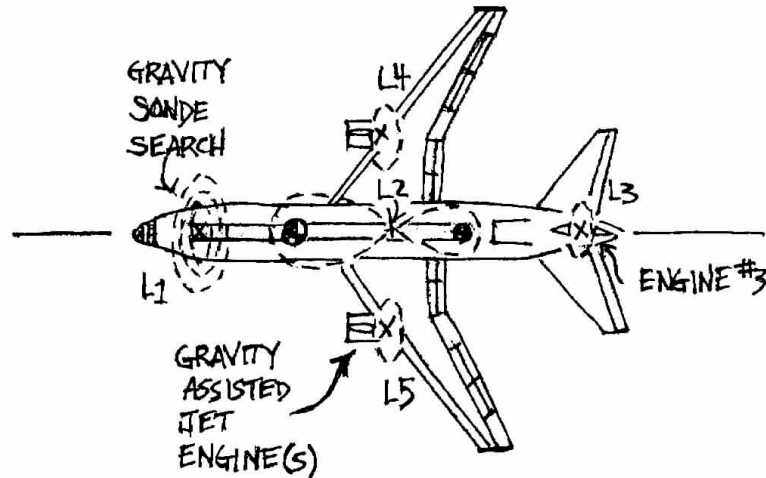
Naturally, this is easier said than done - the mass balance will need to be exact, and since it changes during flight as fuel is used up, the mass balance point will need to change too. Hence, the two "primary masses" in the cargo bays will need to move on tracks, for a continuous computer balance of weight. Not to mention that the zero velocity curves, and the Lagrange points, exist only in a rotating system - and the aircraft moves along a straight line. This is where the nature of gravity developed at length in the Theory comes into play.



Gravity Sonde Installation and Deployment

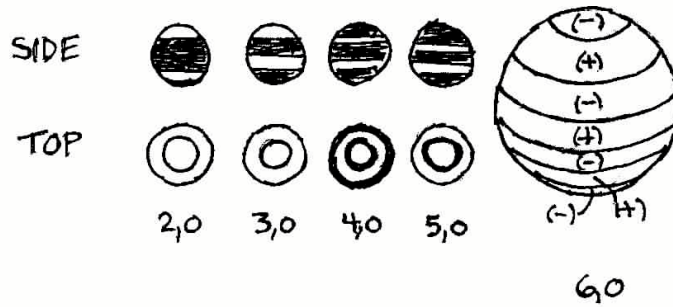
The idea of using "gravity strings" near Earth's surface to facilitate travel is akin to using the jet stream, which may in fact be a side effect of the overall gravity scheme near Earth. Having the aircraft carefully balanced will minimize any spurious readings of local gravity (or at least be able to eliminate it from the readings easily) by a tandem arrangement of gravity sensors similar to the Topex-Poseidon array used to map the Earth's gravity from orbit in space. (Two satellites are attached together along a length of cable, and GPS monitors measure their relative position constantly, enabling gravitational anomalies on the

Earth surface of sizes as small as large oil fields.) Two configurations are possible; parallel and in-line, each tube being a gravity sensor with integral instrumentation.

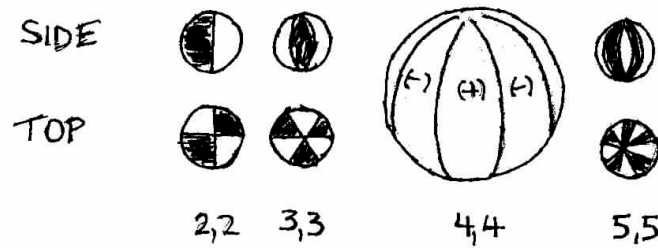


Theoretical Operation of the Gravity Sonde

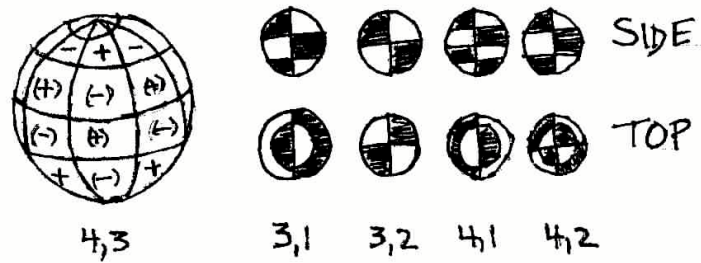
This same configuration may be used to seek gravity strings, perhaps concentrated along the lines dividing the tesseral/sectoral harmonics used to model Earth's gravity field. The figures below show how the Earth's gravity is modeled by a system of quadratures, at several levels. There are thousands of terms, with smaller and smaller segments; which all together allow the Earth's gravity field to be modeled more and more accurately. The mathematics of these huge matrices (which require the largest computers in the world to solve - over five hundred massively paralleled computers, all together) are based on the lines between segments, which typically follow lines of longitude and latitude. Theoretically, these lines are themselves gravity strings, and comprise the overall gravity matrix encompassing the Earth.



Zonal Harmonics



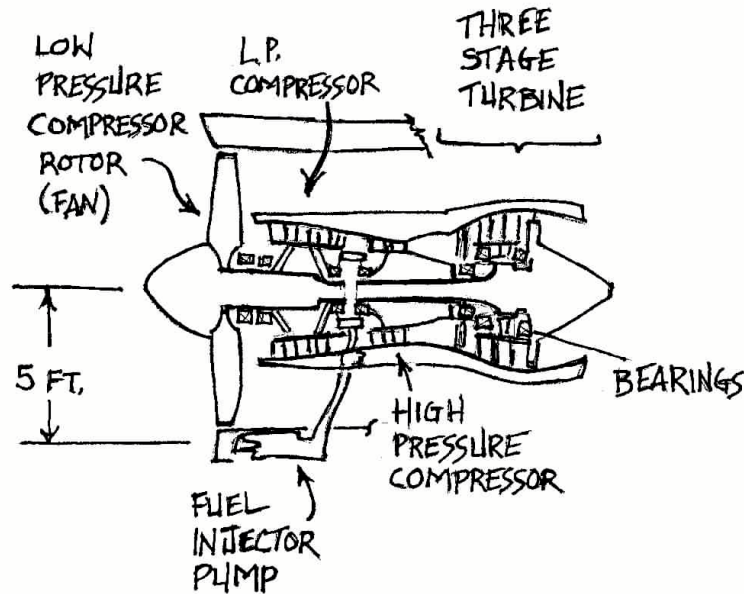
Sectorial Harmonics



Tesseral Harmonics

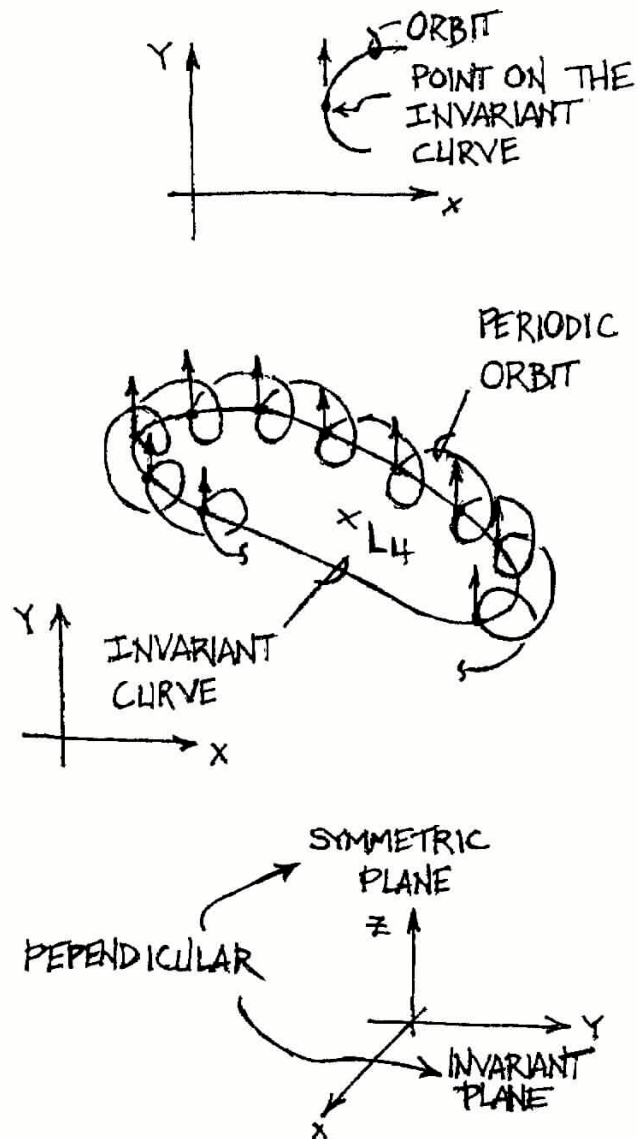
The Tristar design modifications, then, are intended to reduce the gravity profile of the aircraft - the way stealth technology reduces the radar profile of aircraft - so that it can study the gravity field more closely. Ultimately, once the jet can be exactly aligned to a single

gravity string, then the three phase nature of this phenomena postulated in the Theory, can be used to power the jet engines.



Rolls-Royce Turbo Jet Engine

Typically, jet engines have a three phase turbine - for low, medium, and high pressure - and the whole thing rotates because of a fan at the intake - like a turbo charged automobile engine. The Theory shows that the gravity strings have an electromagnetic carrier wave or envelope, and this can be tracked by a proper static charge to the turbine blades. In the context of the 3BP, the following figure shows a periodic orbit around the L4 Lagrange point, which is not unlike the blades of a turbine. Interlacing this with the gravity carrier wave could theoretically allow power to be delivered to the turbines directly from the gravity matrix.



A Complex Periodic Orbit at L4

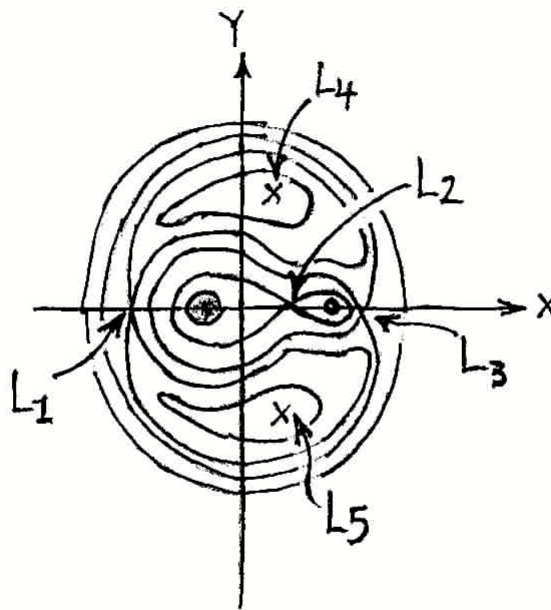
A final compelling illustration, of a particular solution to the "N Body Problem" of Celestial Mechanics, shows a configuration of space very much like what has been talked about here. This particular case is studied toward the end of the Theory, and is a very exciting concept.

A final comment in favor of this Tristar configuration for a deep space aircraft is that it would be capable of atmospheric flight in other worlds, not to mention the notion of space

behaving just like a fluid (e.g. air) at velocities at a high percentage of the speed of light - at which time the jet's control surfaces would be needed, and the jet engines would again track gravity strings and speed the aircraft to its destination.

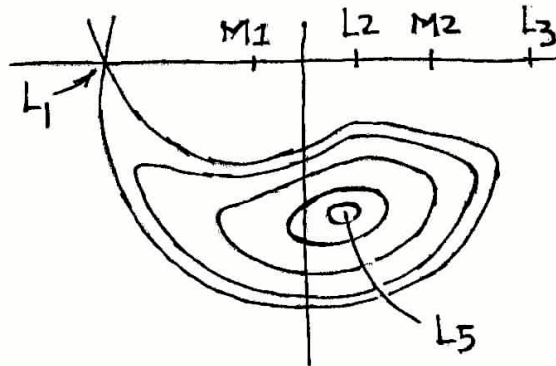
36 The Floor Plan

As might be expected, the starship floor plan, given the shape of its footprint, makes the most efficient use of available space if the various rooms and spaces follow the approximate contours of the outer hull: reminiscent of the zero velocity curves, described in the 3BP Theory section, repeated here.



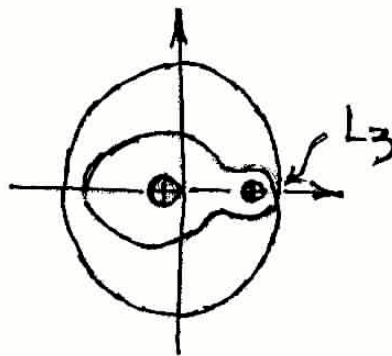
Zero Velocity Curves

These zero velocity curves suggest the optimal location of structural walls. In the theory of orbits, a small object orbiting the two primary masses cannot pass a curve in either side without a gain or loss of energy, e.g. velocity. This is also called an equipotential surface - detailed in the 3BP Theory section - like the contours of terrain, with peaks at L4 and L5. The following figure gives a detail of the relationship between L1 and L5, highlighting a periodic orbit through L1 that also goes around L5. A similar configuration exists for L1 versus L4, because everything is symmetric with these zero velocity curves.



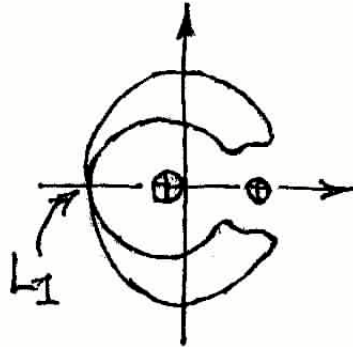
Periodic Orbits around L5

Now, the spacecraft in deep space is a self contained and isolated entity. For all practical purposes, the very distant stars exert a constant force on all sides so that the net force on the vessel is zero. If the mass of the craft is situated to be symmetric with respect to the two masses M_1 and M_2 , then practically speaking the whole vessel acts like a simple 3BP. That's why presumably the two most massive components - the primary and auxiliary "warp drives" are exactly at M_1 and M_2 , and the massive structural walls are also symmetric to them, all together conforming to a planar Three Body Problem. The following two figures show the dynamic relationship between the warp drives, first for full power



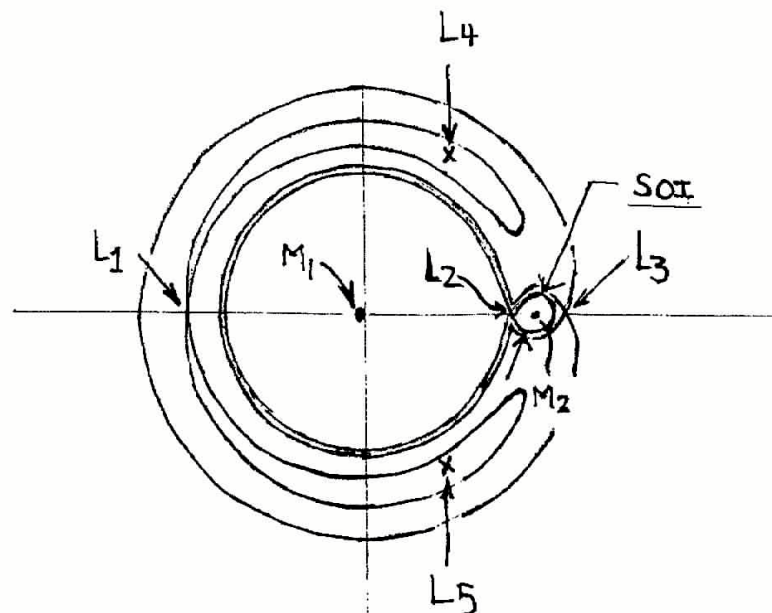
Periodic Orbits Through L3

and then for forward shield protection



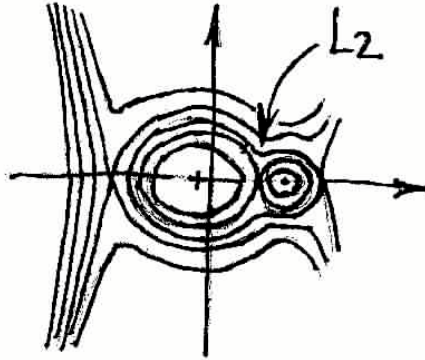
Periodic Orbits Through L1

Even at very high velocities, the spacecraft is a 3BP - there are the stable orbits at L4 and L5; halo orbits at L1, L2 and L3 and a nice closed loop around L2. Each of these form the basis for the power, shields/weapons, and navigation systems - respectively. They all derive their power from the warp drives.



When M_1 is much greater than M_2 ~ e.g. sun & planet

The figure above shows how the zero velocity curves vary as the proportion of mass between M1 and M2 changes (e.g. parallel sections through the 3D "Twin Tower" figure in the 3BP Theory section). This can be used to set up a structural vibration in the structural walls (e.g. M1 and/or M2 rotating about a local center), increasing structural integrity.



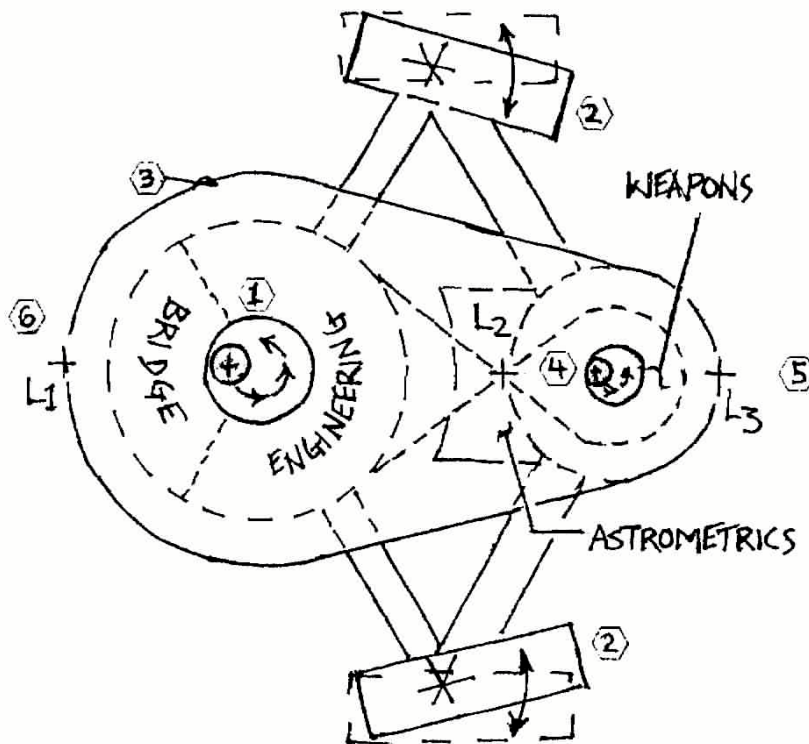
Escape Trajectories from L1

The above figure shows how the forward sensor array is based on the L1 Lagrange Point, similar to the L-1011 Tristar configuration. The key to the navigation system is the free return loop, crossing at the L2 Lagrange point. All navigation data is accessible from this L2 point, potentially assimilated into a real time 3D holograph of the surrounding cosmos.

The other two Lagrange points are important for shields. The forward L1 point is accessible for forward shields, via a halo orbit there. This protects the vulnerable leading edge. Likewise for the L3 Lagrange point to the rear.

37 The Power Systems

The key to forward thrust is to align the spacecraft to a "gravity string" with the nacelles acting as a carrier wave to the gravity helix waves, thus aligning them with the triple helix and allowing the vessel to follow this track - the path of minimum resistance - into deep space.



The Constellation Class Starship

The theory shows how the equilibrium Lagrange points L4 and L5 (the nacelles are there) have a rotational component - e.g. what makes the moon rotate once per revolution around the Earth (keeping the same face always toward Earth, as developed in the Theory), and it is this component that interacts with the gravity string like an electromagnetic pinion gear; moving like a top when you push the central spiral lever down.

Fully extended, the nacelles can lock on to the triple helix gravity wave, and initiate forward momentum. The helix closes as velocity increases, and the nacelles are pulled inward to match the geometry of the gravity string.

The axis of propagation, by Fractal Theory, should be as intricate as the helical waves - three strands for a triplex wave; six for a six-plex wave, and so forth. Generator(s) placed along this axis in the spacecraft will provide a ready source of power while in transit - powering ships systems while keeping the spacecraft aligned with the gravity string itself. (across branch offs, transitions, and terminations)

Assuming the distance between Earth and Alpha Centauri (the nearest star system) requires three transitions, then the axial generators must be capable of twelve phases - three to warp 1, six to warp 2, twelve to warp three - thus all shipboard systems must be designed for twelve phase current. This is quite an efficient system, with minimal harmonic distortion (noise), and a very stable internal ground (like the 12 lunar months per on Earth orbit). This is relatively easy to engineer and, depending on the frequency, is close to direct current.

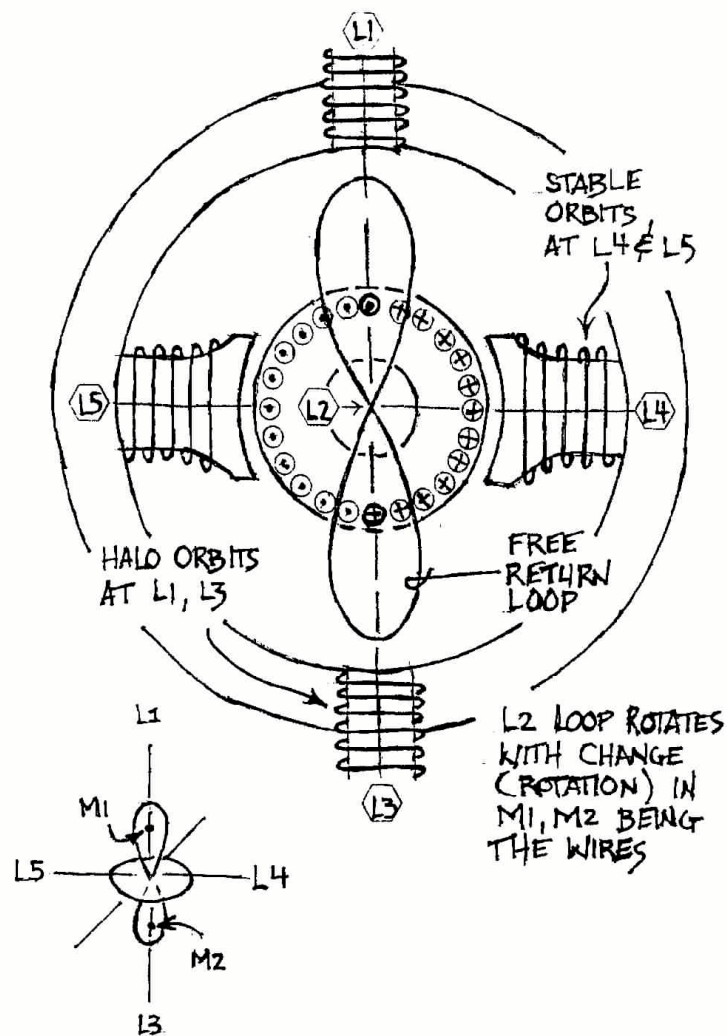
The central power system is fundamentally a twelve phase power system (which can operate three phase devices with slight modification to the field coils). Ideally, electrical devices will be designed specifically for twelve phase operation, which is efficient because there are lower harmonic distortions, and a consistent system ground between all phases.

The Theory shows how the electromagnetic field of motors can be modeled using the principles of Three Body Theory - which means the Lagrange points can be used to make motors and/or generators more efficient.

A similar scheme is developed for the transistor junction diode, which is modeled on the interplanetary trajectory - again, the Lagrange points show how to make semiconductor devices more efficient. In fact, the more sophisticated mosfet devices are optimized using the same techniques used to make the Earth to Mars trajectory more efficient.

The modeling techniques may allow the development of very fast, very efficient electrical power and computing devices. The twelve system is needed so the spacecraft can interact with the gravity matrix. There are further advantages:

- (1) Power energizing a full twelve phase system can be distributed and used efficiently by three phase devices - a single line serving four separate devices or circuits, at three phases each; thus reducing by 25% the wiring needed - or increasing by a factor of four the power that can be transmitted.



Electromagnetic Model for the Lagrange Points

- (2) The field emanated by a twelve phase cable is more uniform than that produced by a three phase system - an important, if subtle, phenomena that is part of the structured matrix. That is, electrical conduit is in the structural walls, which are organized as per the zero velocity curves, and consequently stabilizes the structure indirectly (per the frequency of the power grid).

38 The Gravity Systems

The power drive/nacelle system is designed to align the spacecraft with triax'd helical waves of the gravity string matrix. Consequently, the spacecraft will rotate at a constantly increasing rate as speed increases. This rotation around the centerline of the vessel simulates gravity toward the centerline.

Stabilizing generators along the centerline, which align the spacecraft with the axis of the gravity wave, can have a variable frequency to keep the spacecraft rotating at a constant rate; thus simulating a constant gravitational force.

The spacecraft itself is split in half, with a gently curving arc along the centerline. Outermost spaces, where the gravity increasingly acts more on the walls than the floor make a transition along special decks that spiral/twist across two series of spaces, which are personnel spaces that are built with floors/walls aligned to the gravity field so that, once inside, the space seems natural.

Physically, in the spaces nearest the centerline, people walk on the actual floor - the half deck - but in the outermost spaces from the centerline, people are actually walking on the walls. These outer spaces are mostly storage, acting also as a secondary barrier to any tiny objects that might penetrate the hull. The personnel spaces mid way are, to a much lesser extent, a tertiary barrier - protecting all the vital systems at the center of the vessel. The core is protected from even the most invasive types of radiation, and all hands can gather there in cases of emergency.

39 Artificial Intelligence

The Mars trajectory computer program in the Theory (part Two) illustrates a powerful new computational technique. Virtually every computer program uses some type of numerical integration. In the Mars Pathfinder program, the equations of motion are solved (integrated) using a variable step Runge Kutta integrator.

The optimization algorithm was the integrator to determine neighboring optimal trajectories, making it possible to solve the problem very quickly and accurately. More specifically, the variable step integrator stops whenever all the conditions of the problems change enough to require a new integration calculation (i.e. the step size is varied to match the changing conditions, or forces acting on the spacecraft - close to a large body the steps are very small; far away they are large). Normally this decision must be made by the user, or done randomly; and many trials occur before arriving at a decision. The RK 7/8 makes this decision as part of the program algorithm itself, and using this decision instead of some elaborate, convoluted, artificial decision process is much faster and more efficient.

The gist of the argument is that using the RK 7/8 integrator to make the important decisions not only makes the algorithm more efficient (replacing thousands of computer steps or lines of code, with a single command), more accurate (every computation involves a miniscule round off error, and the cumulative affect of thousands of such steps is great compared to a single step) and greatly compacts the code. That is, the thousand lines of code of Mars Pathfinder replaces a NASA program 100 times larger, finding a better result, faster, and on a much smaller computer - a PC versus a mainframe.

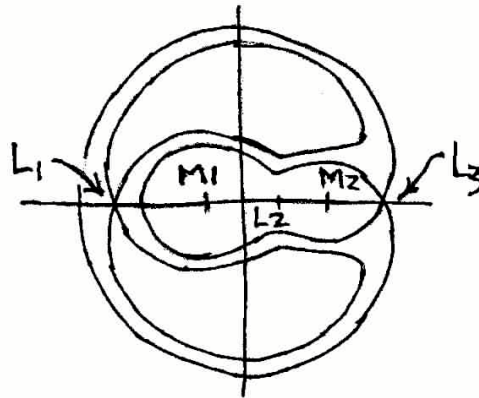
This numerical method is ubiquitous in engineering analysis. Modifying hundreds of such programs will not only reduce the storage space by a factor of 100 or more, but make the routines easier to use and modify. Overall, a complex machine like a starship that could run hundreds of these programs at once - maybe thousands in battle conditions - simultaneously. With the vessel going so fast, needing to make critical decisions each and every instant, then the faster and more compact each routine, the better.

So, speed is critical. Storage space is also at a premium on a space craft. Power is another limiting factor for such a complex, massive computer system - reducing the size by a factor of 100, increases speed of the routine by a like proportion, as well as accuracy and dependability.

40 Weapon Systems

The most practical use for a powerful laser is to vaporize bits of debris in the path of the spacecraft. Even a small speck of dust can cause lots of damage, at such high speeds. Other uses are as a defensive, or offensive, weapon; and as an experimental laser to test the composition of cosmic clouds, comets, or planets for content by studying the spectrum emitted when decomposed.

Given the vast distance that the laser must travel, the beam must be non dissipating. Theoretically, the best way to do that would be to have two particle beams intertwined like a double helix, one proton based and the other electron based; so the two attract and remain intertwined. Small bits of neutral matter can be introduced to increase the power/strength of the laser as needed. Thus, you have a Three Body Problem in cross section, what with the proton, neutron, and the much smaller electron.



A Periodic Orbit through L1 and L3

Such a triplex beam is quite versatile because it has the five Lagrange points as control points - making the beam autonomous in its trek through space, seeking/avoiding mass or charge as the case may be. Otherwise, manipulation of the L1, L2 and L3 points allows a measure of control from the source while the beam is being emitted from the vessel. (You can see why gravity is a triplex'd string.)

The best possible artificial source for this type of laser energy is the homopolar - basically a charged disk rotating extremely fast that when stopped (or slowed down quickly) emits a burst of extremely high voltage. Three such devices, configured as stability drivewheels (two of which are coincident with the warp drives) for the gravity deck, that can be synchronized to emit a pulse along the centerline of the spacecraft.